

Mastering Algebra I:
Course II
Print Activities


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## Table of Contents

## Mastering Algebra I: Course II

## 1 THE REAL NUMBER SYSTEM

### 1.1 Rational \& Irrational Numbers

- Defining the Real Numbers
- Working with Radicals
- The Square Root Function


## 2 POWERS \& POLYNOMIALS

### 2.1 Polynomial Arithmetic

- Working with Powers
- Adding \& Subtracting Polynomial Expressions
- Multiplying Polynomials


### 2.2 Factoring Polynomials

- Finding Common Factors
- Factoring Quadratic Trinomials
- Special Cases


## 3 QUADRATIC FUNCTIONS \& EQUATIONS

### 3.1 Graphing Quadratic Functions \& Equations

- Graphing Parabolas
- Analyzing Properties of Parabolas
- Solving Quadratic Equations by Graphing


### 3.2 Solving Quadratic Equations Using Algebra

- Factoring \& the Zero Product Theorem
- The Square Root Method \& Completing the Square
- The Quadratic Formula

4 ALGEBRAIC EXPRESSIONS \& FUNCTIONS
4.1 Radical Equations \& Functions

- Solving Radical Equations
- The Inverse of the Square Root Function
4.2 Rational Expressions, Equations, \& Functions
- Rational Operations
- Rational Functions
- Rational Equations

5 DESCRIBING DATA
5.1 Graphical Displays

- Stem-\&-Leaf Plots \& Box Plots
- Scatter Plots \& Linear Best-Fit Graphs
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## Notes to the Teacher

Welcome to Destination Math. The student materials in this packet are designed to help students as they progress through the course. These materials, which remain consistent with the philosophy of Destination Math, are specifically intended to:

- keep students focused on the instruction.
- provide students an opportunity to take notes, record information from the program, and reflect on the tutorials.
- allow students an opportunity for additional practice of the instruction in each session.
- provide a more open-ended assessment of the concepts in each session.
- use real-world examples and situations that students can identify with.

There is a set of materials designed to support each session. Each set consists of:

- Student Logbook: This sheet is designed for students to use while viewing the tutorials. It consists of a one-page worksheet where students can record information from the tutorial, take notes, and reinforce their understanding.
- Your Turn: This is a one-page worksheet that provides additional practice for each session. It is designed for students to complete away from the computer to reinforce the concepts they have studied. It may also serve as a guide to what students need to review to complete their mastery of algebraic concepts.

In addition, two sets of materials are provided to cover all the concepts presented in each unit.

- Unit Assessment: The assessment has two pages of problems that cover all skills and concepts in the unit.
- Unit Investigation: This two page activity is designed to explore an algebraic concept that serves as he theme for the unit. It can be used as an initial exploration or as a culminating activity.
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## MASTERING ALGEBRA I: Course 2

MODULE 1: The Real Number System UNIT 1: Rational \& Irrational Numbers

## Defining the Real Numbers

As you work through the tutorial, complete the following questions or sentences.

1. A rational number is a number that is a ratio between two $\qquad$ , as long as the integer in the denominator is not $\qquad$ .
2. A rational number, when expressed as a decimal, either
$\qquad$ or $\qquad$ .
3. The property of density states that between any $\qquad$ numbers, there is always another $\qquad$ _.
4. An irrational number is a number that cannot be expressed as a between $\qquad$ .
5. When expressed as a decimal, an irrational number is neither
$\qquad$ nor $\qquad$
6. The property of density is true for both $\qquad$ and
$\qquad$ numbers.
7. Together, the sets of rational numbers and irrational numbers make up the set of $\qquad$ numbers.

Key Words:
Integer
Whole number Rational number Radical Property of density Irrational number Real number Root

## Learning

## Objectives:

- Define rational numbers.
- Define irrational numbers
- Use the Pythagorean theorem to demonstrate the existence of irrational numbers.
- Approximate the square roots of a set of real numbers and locate them on a number line.

8. The radical symbol, $\sqrt{ }$, expresses the square $\qquad$ of any number.
9. The irrational number $\sqrt{5}$ is expressed exactly when written in
$\qquad$ form.
en ex er
$\qquad$

## MASTERING ALGEBRA I: Course 2 <br> MODULE 1: The Real Number System <br> UNIT 1: Rational \& Irrational Numbers <br> Defining the Real Numbers


a. -6 $\qquad$
$\qquad$
$\qquad$
b. $\frac{1}{5}$ $\qquad$
$\qquad$
$\qquad$
C. $-\frac{8}{3}$ $\qquad$
$\qquad$
$\qquad$
d. $2 \frac{1}{4}$ $\qquad$
$\qquad$
$\qquad$
2. Write each rational number as a decimal. If it is a repeating decimal, use the bar notation to indicate the repeating pattern.
a. $\frac{3}{8}$
a. $\frac{2}{9}$ $\qquad$
c. $\frac{7}{2}$
d. $\frac{5}{7}$ $\qquad$
3. Which rational number lies between 1.22234 and 1.222346 ?
4. Give an example of an irrational number expressed first in its radical form and then in its decimal form. $\qquad$
$\qquad$
5. Approximate the square root of each of the following real numbers to the nearest thousandth. Then plot the approximate locations of the square roots on the number line.
a. $\sqrt{7} \approx$ $\qquad$ b. $\sqrt{35} \approx$ $\qquad$
c. $\sqrt{22} \approx$
d. $\sqrt{14} \approx$ $\qquad$


## MASTERING ALGEBRA I: Course 2 MODULE 1: The Real Number System UNIT 1: Rational \& Irrational Numbers <br> Working with Radicals

## As you work through the tutorial, complete the following questions or sentences.

1. In the formula for the circumference of a circle, $C=2 \pi r$, the irrational number $\pi$ is approximately equal to $\qquad$ _.
2. In the formula for the speed of a tidal wave, $s=3.1 \sqrt{d}$, the value of $s$ can be rational or irrational, depending on the value of $\qquad$ ـ.
3. The expression under a radical symbol is called the $\qquad$ .
4. List the first five non-zero perfect squares: $\qquad$ , $\qquad$
$\qquad$
5. The property of perfect squares states that if $a \geq 0$, then $\qquad$ .
6. The principal square root of a real number is the $\qquad$ -
$\qquad$
$\qquad$ of a number.
7. Complete the statement $\sqrt{a} \times \sqrt{b}=$ $\qquad$ .

Key Words:
Perfect square Principal square root Radical Radicand Rationalize

Learning Objectives:

- Evaluate the square root of a perfect square.
- Simplify the square root of a product.
- Simplify the quotient of two radicals.
- Rationalize the denominator of a radical expression.
- Add or subtract radical expressions using the distributive property.

10. To $\qquad$ means to convert the $\qquad$ of a fraction under a radical sign to a $\qquad$ number.
11. To add or subtract radicals, express the radicals in $\qquad$ form and then combine $\qquad$ radicals.
$\qquad$
$\qquad$

\section*{|  |  |  |
| :--- | :--- | :--- |
| $\square$ |  |  |
|  |  |  |
|  |  |  |
| MASTERING ALGEBRA I: Course 2 |  |  | <br> MODULE 1: The Real Number System <br> UNIT 1: Rational \& Irrational Numbers <br> Working with Radicals}



1. Use the property of perfect squares to complete each expression.

a. $\sqrt{81}=\sqrt{L^{2}}=$ $\qquad$
b. $\sqrt{ }=\sqrt{25}^{2}=$ $\qquad$
c. $\sqrt{\square}=\sqrt{()^{2}}=12$
2. List the first five consecutive perfect squares greater than 100.
$\square$
$\square$
$\square$
3. Express each radical in simplest form.
a. $\sqrt{160}$ $\qquad$ b. $5 \sqrt{108}$ $\qquad$ c. $-\frac{1}{14} \sqrt{490}$ $\qquad$
4. Simplify the product $\left(3 \sqrt{\left.4^{3}\right)}(-2 \sqrt{28})\right.$. $\qquad$
5. Simplify the expression
$\frac{\pi}{3} \sqrt{\frac{8}{32}}$
6. Rationalize each denominator.
a. $\sqrt{\frac{1}{6}}$
b. $\frac{3}{11}$ $\qquad$ c. $\sqrt{\frac{2}{7}}$ $\qquad$
7. Simplify these radical expressions by rationalizing the denominators.
a. $\frac{8 \sqrt{32}}{8 \sqrt{50}}$
b. $\frac{\sqrt{1000}}{\sqrt{8}}$ $\qquad$ c. $\sqrt{\frac{36}{27}}$ $\qquad$
8. Simplify the expression $7 \sqrt{2}+3 \sqrt{18}$. $\qquad$
9. The formula $t=\sqrt{\frac{2 s}{9.8}}$ gives the time, $t$, in seconds that it takes an object at rest to fall $s$ meters, where 9.8 is the acceleration due to gravity, in meters per second squared. How many seconds will it take an object to fall 58.8 meters? Write your answer in simplest radical form. $\qquad$

## H $\quad+\quad$ MASTERING ALGEBRA I: Course 2 MODULE 1: The Real Number System UNIT 1: Rational \& Irrational Numbers <br> The Square Root Function



As you work through the tutorial, complete the following questions or sentences.

1. If $x$ represents the area of a square, then $\qquad$ represents the length of its side.
2. Explain how you know that the square root relationship is not linear.
3. Why is the square root relationship a function? $\qquad$
4. To $\qquad$ is to estimate the value of a function between two known values in the $\qquad$ _,
5. To $\qquad$ is to infer the value of a function in an unobserved interval from values in an already $\qquad$ interval.
6. What set of numbers describes the domain of the square root function?

Key Words:
Square root Domain Range Interpolate Extrapolate Parameter Nonlinear function

Learning Objectives:

- Graph a finite set of ordered pairs $(x, \sqrt{x})$.
- Graph the square root function.
- Identify the domain, range, and equation of the square root function.
- Examine the effect of $a$ on the graph of $y=a \sqrt{x}$.

7. What set of numbers describes the range of the square root function?
8. In the equation $y=3.1 \sqrt{x}$, the coefficient 3.1 is called a $\qquad$ .
9. For graphs of functions in the form $y=a \sqrt{x}$, the value of $a$ affects the ____of the graph and determines the through which the graph passes.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 1: The Real Number System
UNIT 1: Rational \& Irrational Numbers
The Square Root Function


1. Complete the table of values below. Round values of $\sqrt{x}$, to two decimal places.

| $x$ | 0 | 1.3 | 2.0 | 2.7 | 3.4 | 4.8 | 5.2 | 5.9 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\sqrt{x}$ |  |  |  |  |  |  |  |  |

2. Plot the points in the table above on these axes.

3. Graph the square root function $y=\sqrt{x}$ for values of $x$ from 0 to 25 .

4. Identify the graph of each equation by matching the number of the graph to the corresponding equation.
a. $y=0.7 \sqrt{x}$ $\qquad$
b. $y=-\frac{2}{3} \sqrt{x}$ $\qquad$
c. $y=2.4 \sqrt{x}$ $\qquad$
d. $y=3 \sqrt{x}$ $\qquad$

e. $y=-3 \sqrt{x}$ $\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

 MODULE 1: The Real Number System UNIT 1: Rational \& Irrational Numbers
## いnot Assessment

1. Identify each number as rational or irrational. Justify your answer.
a. 0.35621 $\qquad$ b. $\frac{12.5}{7}$ $\qquad$
c. $.5 \pi$
d. 0.552552
e. $-\sqrt{9}$
f. $\sqrt{8}$
g. $12.31 \overline{45}$ $\qquad$ h. 2.121121112 .
2. Is it always, sometimes, or never true that a rational number can be expressed as a terminating decimal? $\qquad$
3. Is it always, sometimes, or never true that an irrational number can be expressed as a nonterminating decimal? $\qquad$
4. In a right triangle with sides of length $a$ and $b$ and $a$ hypotenuse of length $c$, the Pythagorean theorem states that $a^{2}+b^{2}=c^{2}$. Determine whether the length of the hypotenuse of a right triangle whose sides are 4 units and 5 units long is a rational or irrational number. $\qquad$
Explain your answer. $\qquad$
$\qquad$
$\qquad$
5. Each pair of numbers below represents the lengths of the sides of a right triangle. Which pairs represent the legs of a right triangle where hypotenuse has a length that is irrational? Circle your answers.
a. $(12,5)$
b. $(12,13)$
c. $(8,15)$
d. $(2,4)$
6. Plot each irrational number on the number line below.
a. $\sqrt{0.2}$
b. $\sqrt{17}$
c. $\sqrt{3}$
d. $\sqrt{1.4}$

$\qquad$
$\qquad$

## Unio Assessment

7. Which statements below are true if $a \neq b$ and $b \neq 0$ ?

Circle your answer.

a. $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$
b. $\sqrt{a-b}=\sqrt{a}-\sqrt{b}$
c. $\sqrt{a \times b}=\sqrt{a} \times \sqrt{b}$
d. $\sqrt{\frac{a}{b}}=\frac{\sqrt{ } a}{\sqrt{b}}, b \neq 0$
8. Express each radical expression in its simplest radical form.
a. $\sqrt{75}$ $\qquad$ b. $\sqrt{0.0036}$
c. $\sqrt{5} \times \sqrt{85}$ $\qquad$ d. $\sqrt{98 \times 14}$
e. $\sqrt{\frac{96}{2}}$ $\square$ f. $\frac{\sqrt{96}}{\sqrt{8}}$
g. $\sqrt{180}-\sqrt{45}$ $\qquad$ h. $3 \sqrt{12}+4 \sqrt{108}$
i. $\sqrt{\frac{9}{5}}$ $\qquad$ j. $\frac{2}{\sqrt{10}} \times \frac{5}{\sqrt{2}}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
9. Which of the following ordered pairs describe points that lie on the graph of $y=\sqrt{x}$ ? Circle your answers.
a. (1.8, 1.34 )
b. $(2,4)$
c. $(16,4)$
e. $(4.90,24)$

10 Identify the graph of each equation by matching the number of the graph with the corresponding equation.
a. $y=\frac{12}{5} \sqrt{x}$ $\qquad$
b. $y=\frac{3}{2} \sqrt{x}$ $\qquad$
c. $y=\frac{1}{3} \sqrt{x}$
d. $y=-\frac{11}{2} \sqrt{x}$ $\qquad$
e. $y=\frac{7}{4} \sqrt{x}$ $\qquad$


$\qquad$


## MASTERING ALGEBRA I: Course 2

 MODULE 1: The Real Number System UNIT 1: Rational \& Irrational Numbers
## 

## Exploring the View from an Aircraft



When flying on a clear day, you can see all the way to the horizon. However, because the Earth is nearly a sphere, it is not possible to see beyond the horizon even with binoculars or a telescope.

The approximate distance in miles to the horizon is given by the formula $d=1.22 \sqrt{x}$, where $x$ represents the altitude in feet.

1. Graph this function, scaling the horizontal axis to include values from 0 to 35,000 and the vertical axis to include values from 0 to 300 .

2. List the coordinates of five reference points from the graph $\qquad$
$\square$
$\qquad$
$\qquad$
$\qquad$
3. What does the point whose $x$-coordinate is zero represents?
$\qquad$
4. Use the graph to find the approximate distance to the nearest ten, to the horizon from an aircraft at an altitude of 30,000 feet. $\qquad$
$\qquad$

5. Use the graph to find the approximate altitude to the nearest thousand, of
 an aircraft whose distance from the horizon is 100 miles. $\qquad$ .
6. Use the graph to find the approximate altitude to the nearest thousand, of an aircraft whose distance from the horizon is 200 miles. $\qquad$
7. Calculate to the nearest ten, how the distance to the horizon changes as the altitude of the plane decreases from 35,000 feet to 25,000 feet. Show your work.

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials

## UNIT 1: Polynomial Arithmetic

## Working with Powers



As you work through the tutorial, complete the following questions or sentences.

1. An $\qquad$ indicates the number of times a $\qquad$ is used as a factor.
2. Any non-zero number raised to the zero power equals $\qquad$ .
3. $a^{-n}=$ $\qquad$ where $a \not \approx 0$ and $n$ is an integer.
4. A non-zero number raised to a power is equal to the $\qquad$ of the number raised to the $\qquad$ of that power.
5. In the example $10^{11} \times 10^{2}$, you can multiply these two expressions because each factor has the same $\qquad$ _.
6. For any non-zero real number $a, a^{r} \times a^{s}=$ $\qquad$ where $r$ and $s$ are $\qquad$ _.
7. For any non-zero real number $a, a^{r} \div a^{s}=$ $\qquad$ where $r$ and $s$ are $\qquad$ _.

Key Words:
Power
Base
Exponent

Learning
Objectives:

- Simplify expressions containing negative exponents and 0.
- Simplify expressions involving the product and quotient of two powers.
- Simplify expressions involving the power.
- Simplify expressions involving the power of a product and a quotient.

9. For any non-zero real numbers $a$ and $b(a b)^{n}=$ $\qquad$ where $n$ is an $\qquad$ .
10. For any non-zero real numbers $a$ and $b\left(\frac{a}{b}\right)^{n}=$ $\qquad$ where $n$ is an $\qquad$ _.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

## MODULE 2: Powers \& Polynomials

UNIT 1: Polynomial Arithmetic

## Working with Powers

1. Express each of the following numbers as a power of a base that is a
 prime number.
a. 9
b. $\frac{1}{9}$ $\qquad$
c. $3^{5-1}$ $\qquad$ d. $\frac{1}{3}-2$ $\qquad$
2. $85^{\circ}$ is equal to $\qquad$ .
3. Apply the laws of exponents and simplify the following expressions:
a. $b^{-3} \times b^{8}$ $\qquad$ b. $-\left(c^{4}\right)\left(3 c^{-2}\right) c$ $\qquad$
c. $\frac{25^{\circ}}{25^{-6}}$
d. $3^{3} \times\left(3^{2}\right)^{-2}$ $\qquad$
e. $\left(2 x^{3} y^{4}\right)^{5}$ $\qquad$ f. $\left(\frac{5 x}{2 y}\right)^{4}$ $\qquad$
4. The following chart shows approximate distance in Kilometers from the sun to some of the planets in our solar system. Complete the chart using scientific notation.

| Planet | Approximate distance (km) | Distance in scientific notation |
| :--- | :--- | :--- |
| Mercury | $58,000,000$ |  |
| Earth | $150,000,000$ |  |
| Mars | $230,000,000$ |  |
| Saturn | $1,400,000,000$ |  |
| Pluto | $5,900,000,000$ |  |

5. One kilometer is equal to 1,000 or $10^{3}$ meters. Use scientific notation to express the distance in meters from Saturn to the sun. $\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials
UNIT 1: Polynomial Arithmetic

## Adding \& Subtracting Polynomial Expressions



As you work through the tutorial, complete the following questions or sentences.

1. The area of a square with sides of length $x$ is written as $\qquad$ .
2. A monomial in one variable is a term of the form $\qquad$ , where $a$ is a
$\qquad$
$\qquad$ ,$x$ is a $\qquad$ and $n$ is a $\qquad$ integer.
3. A $\qquad$ is a monomial or a finite sum of unique monomials.
4. The polynomial expressions $x^{2}+2 x+1$ is a $\qquad$ . because it is made up of $\qquad$ .
5. When the terms in a polynomial are arranged so that the exponents of the variable decrease from $\qquad$ to $\qquad$ the polynomial is said to be arranged in $\qquad$ .
6. When the terms in polynomial are arranged so that the exponents of the variable increase from $\qquad$ to $\qquad$ the polynomial is

Key Words:
Monomial
Polynomial Binomial
Trinomial
Descending order
Ascending order (opposite) of a polynomial

## Learning

Objectives:

- Explore the definitions related to polynomial expressions.
- Arrange the terms of a polynomial expression in ascending or descending order.
- Find the sum and difference of two (or more) polynomials. said to be arranged in $\qquad$ _.

7. To determine if the sum of two polynomials is correct, substitute a
$\qquad$ for $x$. If you $\qquad$ the expressions and the result is an $\qquad$ then you know the sum is correct.
8. Why are $x^{2}$ and $2 x$ not like terms? $\qquad$ .
9. Fill in each box with an example of each type of expression.

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials
UNIT 2: Polynomial Arithmetic

## Adding \& Subtracting Polynomial Expressions



1. Is $2 x^{-3}$ a monomial? $\qquad$ Explain your answer, using the definition of a monomial. $\qquad$

$\qquad$
$\qquad$
2. Simplify the following polynomial expressions. Then indicate whether the simplified expression is a monomial, a binomial, or a trinomial.
a. $-6+2 x^{2}+9+x-3$ $\qquad$
 $\qquad$
b. $4 s^{23}+15 s-7 s^{17}-16 s$ $\qquad$
$\qquad$
3. Add the following polynomials and Write each sum in descending order.
a. $\left(5 x^{2}-3 x+7\right)+\left(2 x^{3}+5 x^{2}+x+5\right)$ $\qquad$
b. $\left(-3 b^{4}+b^{2}-b\right)+\left(b^{4}-b^{2}+4\right)$
c. $\left(9 c^{2}+3 c-2\right)+\left(7 c^{3}-3 c^{2}-3 c\right)$ $\qquad$
4. Subtract the following polynomials and Write each difference in ascending order.
a. $\left(7 a^{3}-a\right)-\left(-4 a^{3}+2 a\right)$ $\qquad$
b. $\left(8 x^{3}-2 x^{2}+1\right)-\left(4 x^{2}+x-2\right)$ $\qquad$
c. $\left(b^{2}+b-4\right)-\left(b^{3}-2 b^{2}-4\right)$ $\qquad$
5. The center pane in a panel of three windows has an area represented by the trinomial $2 n^{2}+5 n+3$. Each side pane in the panel has an area represented by the binomial $n^{2}+2 n$.
a. What is the total area of the two side panes in terms of $n$ ? $\qquad$
b. What is the area of the center pane and one side pane in terms of $n$ ?
$\qquad$
c. What is the total area of the panel of three windows in terms of $n$ ?
$\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials UNIT 1: Polynomial Arithmetic

## Multiplying Polynomials



As you work through the tutorial, complete the following questions or sentences.

1. The width of the front of the brochure is represented by the binomial
$\qquad$ _.
2. For the front cover of the brochure, the expression that demonstrates the application of the $\qquad$ property to $(n+10)(n+1)$ is
$\qquad$ .
3. The result of multiplying two binomials, is the $\qquad$ of four
$\qquad$ _.
4. What do the letters in FOIL represent when multiplying two binomials? F $\qquad$
O $\qquad$
I $\qquad$
L $\qquad$
5. To check that a product is correct, $\qquad$ a value for the variable and see if the result is an $\qquad$ .
6. For all real numbers $a$ and $b,(a+b)^{2}=$ $\qquad$ .
7. For all real numbers $a$ and $b,(a-b)^{2}$ is equal to the trinomial

Key Words:
Product
Factor Binomial
Trinomial
Perfect square trinomial

## Learning

Objectives:

- Use an area model to represent the product of two binomials.
- Use the distributive property to find the product of two polynomials.
- Recognize the square of a binomial as a perfect square trinomial.
- Recognize the product of the sum and difference of two monomials as the difference of two squares.

8. For all real numbers $a$ and $b,(a+b)(a-b)$ is equal to
$\qquad$ —.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

## MODULE 2: Powers \& Polynomials

UNIT 1: Polynomial Arithmetic
Multiplying Polynomials


1. Express the area of each rectangle in this diagram as the product of its length and width, and as a trinomial in terms of $n$.

a. Rectangle $A B D C$ $\qquad$ $=$ $\qquad$
b. Rectangle CDFE $\qquad$ $=$ $\qquad$
c. Rectangle EFHG $\qquad$ $=$ $\qquad$
d. Rectangle $A B H G$ $\qquad$ $=$ $\qquad$
2. Use the FOIL method to multiply and simplify $(n+3)(4 n-2)$.
3. Find the square of the following binomials. Verify that your answers are correct by substituting -2 for the variable.
a. $(3 b+2)^{2}$ $\qquad$
b. $(5 y+3)^{2}$ $\qquad$
4. The product of $(x+4)(x-4)$ is $\qquad$ .
5. The length of one panel of a greeting card is $n$. The width is $n+8$. If the greeting card consists of two panels of equal size, what is the area of the card in terms of $n$ ? $\qquad$ Explain how you arrived at your answer. $\qquad$
$\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials UNIT 1: Polynomial Arithmetic

## Uniot Assessment

1. Which of the following expressions are equivalent to $\frac{1}{4^{-2}}$ ? $\qquad$

a. $4^{-2}$
b. $4^{2}$
c. 16
d. $\frac{1}{8}$
2. Simplify each of the following expressions.
a. $-\left(8 a^{-6}\right)\left(4 a^{9}\right)$ $\qquad$
b. $\frac{15 \mathrm{r}}{3 \mathrm{r}^{-5}}$ $\qquad$
c. $4^{5} \times\left(4^{3}\right)^{-3}$ $\qquad$
d. $\left(-2 x^{0} y^{3}\right)^{4}$ $\qquad$
e. $\left(2 s^{n+2}\right)^{3}$ $\qquad$
f. $\left(\frac{4 r}{7_{s}}\right)^{3}$ $\qquad$
3. An adult spider mite is typically 0.038 inch long. Express this number in scientific notation. $\qquad$
4. The following trinomials represent the areas of three rectangular floor mats.

Mat A: $4 n^{2}+11 n-3$
Mat B: $3 n^{2}-n-2$
Mat C: $2 n^{2}+14 n+12$
a. What polynomials represents the area of the floor covered by Mats A and $B$ ? $\qquad$
b. What polynomials represents how much more of the floor is covered by Mat C than Mat A? $\qquad$
c. What polynomials represents how much more of the floor is covered by Mat B than Mat C? $\qquad$
5. Use the FOIL method to multiply and simplify $(2 n+3)(3 n-4)$
$\qquad$ —.
$\qquad$
$\qquad$

6. The product of $(x-5)(x+5)$ is $\qquad$ .
a. $x^{2}+10 x-25$
b. $x^{2}-10 x-25$
c. $x^{2}+25$
d. $x^{2}-25$
7. Verify that the answer choice you selected for Question 6 is correct by letting $x$ equal 3 . Show your work.
8. The diagram below represents a rectangular garden with a rectangular fountain in the center. Complete the following statement, expressing your answers in terms of $n$.

a. A binomial that represents the area of the fountain is $\qquad$ .
b. A trinomial that represents the total area needed for the flowers and the fountain is $\qquad$ -.
c. A trinomial that represents the area where flowers can be planted is $\qquad$ -.

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials UNIT 1: Polynomial Arithmetic

## Unoif Onvestigaxfon

## Investigating Area



A developer plans to build a new shopping mall. The mall will occupy a large rectangular area, with a center atrium with a walkway and sitting area for shoppers. Three department stores have already committed to rent space in the mall. Design a mall based on the following specifications.

- The length of the mall is three times the width of the mall.
- The minimum area that can be used for the mall is 30,000 square feet.
- The maximum area that can be used for the mall is 120,000 square feet.
- Carlyle \& Company is the largest store
- The other two department stores have the same area, but their dimensions are different, and they are smaller than Carlyle \& Company.
- The area of the rectangular atrium is equal to one half the area of Carlyle \& Company.
- The mall must have a minimum of six stores in order to cover development costs.
- The maximum number of stores than can occupy the mall space is 10 .
- The parking area must have one level of parking for every 10,000 square feet of space used for stores.

1. Complete the diagram below to illustrate your mall design.

$\qquad$

## 

2. Using the variable $m$, list the algebraic expressions for the dimension and area of each following.

a. Carlyle \& Company Length $\qquad$ Width $\qquad$ Area $\qquad$
b. Department store 1 Length $\qquad$ Width $\qquad$ Area $\qquad$
c. Department store 2

Length $\qquad$ Width $\qquad$ Area $\qquad$
d. Atrium Length Length $\qquad$ Width $\qquad$ Area $\qquad$
e. Other stores in the mall:
$\qquad$
$\qquad$
$\qquad$
$\qquad$
3. Decide on the area of your mall within the given limits. $\qquad$
a. Based on the area, what are the dimensions of the mall?

Length $\qquad$ Width $\qquad$
b. How many parking levels will your mall need?
4. The construction costs per square foot for store and atrium space is $\$ 50$.
a. What will it cost to build the mall you designed? $\qquad$
b. If the initial budget allocated for construction is $\$ 5,000,000$, can you construct your mall? $\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials UNIT 2: Factoring Polynomials

Finding Common Factors


As you work through the tutorial, complete the following questions or sentences.

1. A prime number is a $\qquad$ with exactly two factors: $\qquad$ and $\qquad$ .
2. Why is 1 not a prime number?
3. A positive integer that is neither prime nor equal to 1 is a
$\qquad$
$\qquad$ .
4. To find the nearest common factor of two numbers, match the
$\qquad$ that the numbers have in common and then find the $\qquad$ of these numbers.
5. To find the greatest variable common factor of two monomials having the same base, $\qquad$
$\qquad$ _.
6. The exponent of the variable in a monomial term in one variable is the
$\qquad$ of the monomial term.
7. What is the greatest common factor of $24 n^{3}$ and $60 n^{2}$ ? $\qquad$
8. To factor a polynomial means to express it as $\qquad$
Key Words:
Factor
Prime number Composite number Greatest common factor
Degree of a monomial
Degree of a polynomial
Prime polynomial
Common monomial factor
Fundamental theorem of arithmetic

Learning Objectives:

- Discriminate between prime and composite numbers.
- Identify the greatest common monomial factor of two (or more) monomials.
- Factor a polynomial by finding its greatest common monomial factor.
- Factor a polynomial by finding a common binomial factor.

9. The degree of a polynomial is the $\qquad$ degree of the $\qquad$ terms within it.
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials
UNIT 2: Factoring Polynomials
Finding Common Factors


1. Write the prime factorization for each of the following monomial.

a. 60 $\qquad$
b. $155 x y$ $\qquad$
c. $144 n^{2}$ $\qquad$
2. Find the greatest common monomial factor of each of the following groups of monomial expressions.
a. $72 y^{4}, 40 y^{3}$ $\qquad$
b. $4 a^{5},-12 a^{4}, 28 a^{3}$ $\qquad$
3. Consider the polynomial $6 x^{2}+3 x$.
a. Draw or use algebra tiles such as $x$ and $x^{2}$, as shown on the left, to make a geometric representation of the rectangular area represented by the polynomial.

b. Use your drawing to express the polynomial as the product of two polynomials.
c. Verify that the product of the two polynomial factors represents
$6 x^{2}+3 x$ by substituting 4 for $x$.
4. Factor the following polynomials completely.
a. $12 n^{3}+20 n$
b. $72 y^{4}+40 y^{3}$ $\qquad$
c. $x^{2}+2 x+5 x+10$
d. $3 m^{2}+21 m+6 m+42$ $\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials
UNIT 2: Factoring Polynomials

## Factoring Quadratic Trinomials



## As you work through the tutorial, complete the following questions or sentences.

1. When factoring the trinomial $x^{2}+10 x+24$, you are looking for a pair of numerical factors whose product is $\qquad$ and whose sum is $\qquad$ .
2. What are the binomial factors for $x^{2}+10 x+24$ ? $\qquad$
Key Words:
Binomial
Trinomial
Quadratic term
Linear term
Constant term
Standard form of a quadratic expression
3. A monomial term whose degree is 2 is called a $\qquad$
4. A $\qquad$ is a monomial term whose degree is 1 .

5 A $\qquad$ is another name for a monomial term whose degree is 0 .
6. Is the quadratic expression $x^{2}+10 x+24$ written in standard form?
$\qquad$ Explain your answer. $\qquad$
Learning
Objectives:

- Factor a quadratic trinomial of the form $1 x^{2}+b x+c$, where $c>0$.
- Factor a quadratic trinomial of the form $1 x^{2}+b x+c$, where $c>0$.
- Factor a quadratic trinomial of the form $a x^{2}+b x+c$, where $a=1$.

7. If the constant in a quadratic polynomials that can be factored is negative, then the signs of the constants in the binomial factors are
$\qquad$ _.
8. What are the binomial factors of $y^{2}+7 y-12$ ? $\qquad$
9. Represent the quadratic expression $2 r^{2}+7 r+6$ as the product of two binomial factors $\qquad$ .
10. Represent the quadratic expression $6 n^{2}+11 n-10$ as the product of two binomial factors. $\qquad$
$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials
UNIT 2: Factoring Polynomials

## Factoring Quadratic <br> Trinomials

1. Consider the trinomial $2 g^{2}+9 g+10$, $\qquad$

a. Draw or use algebra tiles to complete a rectangle whose area is $2 g^{2}+9 g+10$.

b. Use the model to express the trinomial as the product of two binomials
c. Check your factors by substituting 2 for $g$ in the factors and the original polynomial.
2. Given the trinomial $5 s+s^{2}+1$ :
a. Rewrite the quadratic expression in standard form. $\qquad$
b. Name the quadratic term. $\qquad$
c. Name the linear term.
d. Name the constant term. $\qquad$
3. Factor each of the following polynomials completely.
a. $x^{2}+5 x+6$ $\qquad$
b. $d^{2}-4 d-32$ $\qquad$
c. $2 p^{2}+7 p+3$ $\qquad$
d. $3 y^{2}-7 y+4$ $\qquad$
e. $3 f^{2}-3 f-18$ $\qquad$


## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials UNIT 2: Factoring Polynomials

## Special Cases

## As you work through the tutorial, complete the following questions or sentences.

1. If $a$ trinomial is of the form $a^{2}+2 a b+b^{2}$, then it is a perfect square and is equal to $\qquad$ .
2. The polynomial $\left(a^{2}-b^{2}\right)$ is known as the $\qquad$ of
$\qquad$
$\qquad$ _.
3. What are the binomial factors of $4 x^{2}-9$ ? $\qquad$
4. If $a$ and $b$ are real numbers, $a^{2}-b^{2}=$ $\qquad$
5. Factor $25 k^{2}-144$. $\qquad$
6. Is the polynomial $x^{4}-64$ an example of the difference of two squares?
$\qquad$ Explain your answer. $\qquad$
7. Factor $x^{4}-64$ completely $\qquad$
8. A polynomial that cannot be factored is $\qquad$
9. When factoring a polynomial expression, use the following quidelines.
a. First, always check for any $\qquad$ _____ and apply the $\qquad$ property to simplify the expression.
b. Then, look at the remaining polynomial to see if you recognize a pattern, such as a $\qquad$
$\qquad$ or the $\qquad$
$\qquad$
$\qquad$ .
10. If $a$ and $b$ are real numbers and have no common factors then $a^{2}+b^{2}$ is
$\qquad$ _.
$\qquad$

## MASTERING ALGEBRA I：Course 2 MODULE 2：Powers \＆Polynomials UNIT 2：Factoring Polynomials <br> Special Cases

1．Complete the table below．

| Factored form | Trinomial expression | Special case |
| :--- | :--- | :--- |
|  | $x^{2}+18 x+81$ |  |
| $(2 x+10)^{2}$ |  |  |
|  | $x^{2}-6 x+9$ |  |
|  | $x^{2}-25$ |  |
| $(x+7)($ | $x^{2}-$ | difference of two squares |
|  | $x^{2}+81$ | sum of two squares |
|  | $4 x^{2}-80 x+400$ |  |

2．a．What is meant by the＂square of the difference＂and the＂difference of squares？＂Give an example of each．
$\qquad$
$\qquad$
$\qquad$
b．Use numerical substitution to verify that the examples you gave in（a） are not equivalent．
$\qquad$
$\qquad$
$\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials UNIT 2: Factoring Polynomials

## Unî́ Assessment

1. In the first step of his technique to find prime numbers, Erastosthenes began by
 eliminating all numbers greater than 2 that were multiples of 2 . Describe the additional steps. Erastosthenes used to identify the other prime numbers less than 100.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
2. Factor the number 24 into its prime factors. $\qquad$
3. a. Find two numbers whose greatest common factor is a composite number
$\qquad$ and $\qquad$
b. Find two numbers whose greatest common factor is a prime.
$\qquad$ and $\qquad$
4. What is the greatest common factor of $b^{4}-b^{7}$ ? $\qquad$
5. List the greatest common factor(s) of the terms below.

| Terms | Greatest common factor |
| :--- | :--- |
| 16,24 |  |
| $64 m, 32 m, 96 m$ |  |
| $42 x^{2}, 18 x^{3}$ |  |

$\qquad$

## Unoij Assessment

6. a. Use he following set of algebra tiles to write an identity showing that the product of two binomials is equal to a trinomial.
b. Check your answer using numeric substitution.

$$
x
$$


$\qquad$
$\qquad$
7. Factor each of the following
a. $g^{2}-10 g+2 g-20$ $\qquad$
b. $k^{2}+12 k+36$ $\qquad$
c. $p^{2}-6 p-16$ $\qquad$
d. $4 x^{2}+16 x+15$ $\qquad$
e. $y^{2}+64$ $\qquad$
f. $16 a^{2}-25$ $\qquad$
8. A shop makes rectangular picture frames of different sizes. A standard frame is 2 inches wide. The expressions below represent the dimensions in inches of four rectangular unframed pictures. Find a factored expression for the area of the frame around each picture.
a. $p$ by $p$
b. $h$ by 12

c. $d$ by $2 d$
d. $x^{2}$ by $x^{2}-12$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 2: Powers \& Polynomials UNIT 2: Factoring Polynomials

## Oఐit 0mvestigexfon

## Investigating Floor Plans



A person wants to install a carpet in a square room whose dimensions are $s$ feet by $s$ feet. Along two adjacent sides of the room, the owner wants 2 feet of tile instead of carpet.

1. Let $C$ represent the length of the carpet in feet, then Make a sketch of the floor plan of the room. Label the parts that are to be carpeted and the parts than are to be tiled.
2. Label the sketch above in terms of $C$ and 2 .
3. Use the tiles to help you write a polynomial in expanded form and in factored form that represents the total area of the square region to be tiled and carpeted.

Expanded form $\qquad$

Factored form $\qquad$
4. Suppose the owner wants the same arrangement of carpeting but wants $n$ feet of tile instead of 2 feet.
a. In the space above, represent the new floor plan and label all parts in terms of $c$ and $n$.
b. Write a polynomial in expanded form and in factored form that represents the new area of the tiled region and the carpeted region.

Expanded form $\qquad$

Factored form $\qquad$
$\qquad$

## 

5．What special case of polynomial is represented when the width of the tile on two walls is equal？
$\qquad$

6．Draw an example of a rectangular floor with carpeting and tile whose total area equals the product of two binomials．

7．Represent the total area of the floor，including its carpet and tiles as binomials in expanded form and factored form．

Expanded form $\qquad$

Factored form $\qquad$

8．What is the advantage to factoring the polynomial that represents the area of a rectangular floor． $\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 1: Graphing Quadratic Functions \& Equations

## Graphing Parabolas



## As you work through the tutorial, complete the following questions or sentences.

1. What is a quadratic function? $\qquad$
2. In the function $y=a x^{2}$, the $\qquad$ can be any real number, and the $\qquad$ must be greater than or equal to zero.
3. When the coefficient $a$ in $y=a x^{2}$ is positive, the parabola is concave
$\qquad$ -.
4. When the coefficient $a$ in $y=a x^{2}$ is negative, the parabola is concave
$\qquad$ -.
5. What is the minimum of a parabola whose equation is of the form $y=a x^{2}$. $\qquad$
6. What is the maximum of a parabola whose equation is of the form $y=a x^{2}$. $\qquad$
7. An $\qquad$
$\qquad$
$\qquad$ is a line that divides a shape so that, when folded, the two sides of the shape coincide.
8. What is the equation of the axis of symmetry for a parabola whose equation is of the form $y=a x^{2}$ ? $\qquad$
9. The intersection of a parabola and its axis of symmetry is called the
$\qquad$ of a parabola.


Key Words:
Quadratic function Parabola Parabolic function Minimum of a parabola Maximum of a parabola Symmetry Axis of symmetry Vertex of a parabola Even function

## Learning

Objectives:

- Recognize that the graph of the quadratic equation $y=a x^{2}$ is a function.
For a graph of $y=a x^{2}$ :
- Identify the domain and range.
- Describe the effect of the parameter a on the shape of the graph.
- Determine the minimum and maximum.
- Determine the equation of the axis of symmetry.
- Determine the coordinates of the vertex.
$\qquad$
$\qquad$

| $\square$ |
| :--- |


a. $y=3 x^{2}$
b. $y=2 x$
c. $d=16 x t^{2}$
2. Determine whether the parabolas represented by the following equations are concave up or concave down.
a. $y=-8 x^{2}$ $\qquad$
b. $d=6 t^{2}$ $\qquad$
c. $a=25(-8) b^{2}$ $\qquad$
3. a. Which of the parabolas in Question 2 have a minimum? $\qquad$
b. Which of the parabolas in Question 2 have a maximum? $\qquad$
4. Which of the parabolas in Question 2 rises or falls most steeply? $\qquad$
5. Construct a graph of the function $y=2 x^{2}$, and then answer the questions below. $\qquad$

a. What is the domain and range of this function? $\qquad$
b. What is the equation of the axis of symmetry? $\qquad$
c. What are the coordinates of the vertex of this parabola?
d. Is the parabola concave up or concave down?
e. What is the minimum or maximum of the parabola? $\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 1: Graphing Quadratic Functions \& Equations

## Analyzing Properties Of Parabolas



## As you work through the tutorial, complete the following questions or sentences.

1. What does the constant 1,000 in the equation $y=-4.9 x^{2}+1,000$ represent? $\qquad$
2. The parabola defined by the quadratic equation $y=-4.9 x^{2}+1,000$ is concave $\qquad$ and has a $\qquad$ whose $y$-coordinate is
$\qquad$ .
3. The standard form of a quadratic function in two variables is $y=a x^{2}+b x+c$, where $a \neq \ldots$, and $a, b$, and $c$ are $\qquad$
$\qquad$ _.
4. The equation $h=-4.9 t^{2}+v t$ is an $\qquad$ quadratic equation in two variable $s$ because the constant $c$ is equal to $\qquad$ -.
5. To find the maximum of the parabola $h=-4.9 t^{2}+44.1 t$, first find the
$\qquad$ between the horizontal intercepts, then substitute that value of $t$ into the equation to find the value of $\qquad$ -.
6. The maximum of the parabola whose equation is $h=-4.9 t^{2}+68.6 t$ is
$\qquad$ _.
7. The maximum of the parabola whose equation is $h=-4.9 \dagger^{2}+68.6 \dagger$ occurs when $t=$ $\qquad$ .
8. If $b=0$, then the graph of $y=a x^{2}+c$ is $a$ $\qquad$ whose axis of symmetry is the $\qquad$ and whose $\qquad$ is $(0, c)$.
9. If $c=0$, then the graph of $y=a x^{2}+b x$ has a $y$-intercept of 0 and two
$\qquad$ one of which is always $\qquad$ .

Key Words:
Quadratic function
Parabola
Parabolic function
Minimum of a parabola
Maximum of a parabola Symmetry Axis of symmetry Vertex of a parabola Standard form of a quadratic equation in two variables

Learning
Objectives:

- Examine the properties of parabolas whose equations are of the form $y=a x^{2}+c, c \neq 0$.
- Examine the properties of parabolas whose equations are of the form $y=a x^{2}+b x$.
- Examine the properties of parabolas whose equations are of the form
$y=a x^{2}+b x+c$, $b \neq 0, c \neq 0$. - $\qquad$ .
$\qquad$
$\qquad$


## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 1: Graphing Quadratic Functions \& Equations

## Analyzing Properties Of Parabolas

1. What is the vertical $x$-intercept for each of the following parabolas?

a. $y=3 x^{2}-8 x-5$ $\qquad$
b. $h=4.9 t^{2}+16 t$ $\qquad$
c. $d=-4.9 t^{2}+67$ $\qquad$
2. Which of these equations represent parabolas that have $x=0$ as the axis of symmetry? $\qquad$
a. $y=-8 x^{2}+2 x+16$
b. $h=4.9 t^{2}$
c. $d=24.9 t^{2}+125$
d. $h=4.9 t^{2}+2 \dagger$
3. Use the axes to the right to sketch a parabola hat has the following properties: it is concave up, has an axis of symmetry whose equation is $x=-2$, and has a minimum of -3 .

4. What is the vertex of a parabola that is concave down if the equation of its axis of symmetry is $x=8$, and its maximum is 15 ? $\qquad$
5. A stone is dropped over the edge of a cliff. The distanced between the stone and the ground at any given time tcan be calculated using $d=4.9 t^{2}+400$.
a. What is the maximum of the parabola represented by the equation $d=4.9 t^{2}+400$ ? $\qquad$ .
b. What does the maximum represent?


## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations

## UNIT 1: Graphing Quadratic Functions \& Equations

## Solving Quadratic Equations by Graphing



As you work through the tutorial, complete the following questions or sentences.

1. The flight path of an object that is struck or thrown is called its
$\qquad$ —.
2. Adding the constant -6 to the right side of the equation $h=-0.036 d^{2}+1.29 d$ does not affect the $\qquad$ and
$\qquad$ terms of the equation; it only changes the vertical intercept of the graph from $\qquad$ to $\qquad$ .
3. The axis of symmetry intersects the parabola at its $\qquad$ _.
4. A $\qquad$ or $\qquad$ off an equation is a number that satisfies the equation when it is substituted for a variable.
5. The $\qquad$
$\qquad$ of a function are roots of the corresponding equation when the value of the second coordinate is equal to zero.
6. If a quadratic function has two x-intercepts, then the corresponding quadratic equation when set equal to zero has two $\qquad$
$\qquad$ _.
7. What is the maximum of the parabola whose equation is $h=-0.036 d^{2}+1.29 d-6$, rounded to the nearest tenth? $\qquad$
8. If a quadratic function has only one x-intercept, then the corresponding quadratic equation when set equal to zero has exactly one $\qquad$ .
9. If a quadratic function has no $\qquad$ then the corresponding quadratic equation when set equal to zero has no real solution.

Key Words:
Quadratic function Trajectory
Standard form of a quadratic equation in one variable
$x$-intercept of a graph
Solutions(s) of a quadratic equation in one variable
Root of an equation

Learning
Objectives:

- Discover that the maximum number of real solutions of a quadratic equation is 2 .
- Recognize the relationship between the number of $x$ intercepts of a parabola $y=a x^{2}+b x+c$ and the number of real solutions of the corresponding quadratic equation $y a x^{2}+b x .+$ $\mathrm{c}=\mathrm{O}$.
$\qquad$
$\qquad$


## MASTERING ALGEBRA I：Course 2

MODULE 3：Quadratic Functions \＆Equations
UNIT 1：Graphing Quadratic Functions \＆Equations

## Solving Quadratic Equations by Graphing

1．During a softball game，the pitcher throws the ball to the catcher．The catcher misses the ball，and it hits the ground at a horizontal distance of 25 feet from where it was thrown．The ball＇s trajectory can be represented by the equation $h=-0.06 d^{2}+1.3 d+5$ ，where $h$ is the height of the ball in feet，and $d$ is the distance in feet of the ball from where it was thrown．
a．How high above he ground was the ball when it was thrown？ $\qquad$
b．What was the maximum height of the ball？ $\qquad$
c．Graph the parabola whose equation is $h=-0.06 d^{2}+1.3 d+5$ ．What portion of this graph corresponds to the trajectory of the ball？（Hint： Remember that distance，$d$ ，is always nonnegative．） $\qquad$

2．Analyze the equations below to determine
 whether the parabola corresponding to each equation has zero，one or two horizontal intercepts；whether it is concave up or down；and if it has a maximum or a minimum．

| Equation | Horizontal <br> Intercepts | Concavity | max／min |
| :--- | :--- | :--- | :--- |
| $h=0.5 d^{2}+1$ |  |  |  |
| $y=-3 x^{2}+6 x+1$ |  |  |  |
| $d=-1.9 \dagger^{2}$ |  |  |  |
| $y=4 x^{2}+4 x-35$ |  |  |  |


$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 1: Graphing Quadratic Functions \& Equations
3. Given the equation $y=3 x^{2}+5 x-7$.

a. What is the quadratic term? $\qquad$
b. What is the linear term? $\qquad$
c. What is the constant term? $\qquad$
d. How can you tell if an equation represents a parabola? $\qquad$
e. Which part of a quadratic equation affects the steepness of the parabola? $\qquad$
f. What part of equation determines whether the corresponding parabola is concave up or concave down?
g. What part of the equation is the $y$-intercept of the parabola? $\qquad$
2. Determine whether the parabolas that correspond to each equation have a minimum or a maximum.
a. $h=-4.9 t^{2}$ $\qquad$
b. $\qquad$ $y=5 x^{2}-2 x-4$
c. $h=0.5 d^{2}+1.2 d+2$ $\qquad$
3. Match each parabola below with its corresponding equation:

$$
y=x^{2}-1, y=-x^{2}+1
$$


a.
b.


$\qquad$
$\qquad$

## Unoif Assessment

4. Analyze the equations of these parabolas to determine the number of real solutions of the corresponding quadratic equations set equal to 0 .
a. $d=2 t^{2}+6 t$ $\qquad$
b. $y=5 x^{2}+17$ $\qquad$
c. $h=4.9 t^{2}+5 t-6$ $\qquad$
d. $y=x^{2}$ $\qquad$

A car accelerates to merge onto the highway. If the acceleration remains constant, the distance the car has traveled after any given time can be found using the equation $d=2 t^{2}+v t$, where $t$ is the time in seconds, $v$ is the initial velocity in meters per second, and $d$ is the distance in meters.
5. Graph the function $d=2 t^{2}+4 t$, with + on the horizontal axis, and $d$ on the vertical axis.
6. Suppose a car enters the highway with an initial velocity of 4 meters per second and accelerates for 5 seconds. What is the domain of the function that represents the car's motion during this time period?

7. What is the range of the function during the 5 -second interval?

$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 1: Graphing Quadratic Functions \& Equations

## Investigating Parabolic Motion

Onit Omvestigation
1.Two students are studying the motion of falling bodies. One student drops a ball from the top of a wall that is 50 feet above the ground. The second student throws a ball from the top of the same wall. The ball that was thrown has an initial speed of 10 meters per second. The tables below list the heights $h^{\prime}$ and $h^{2}$ above the ground for each ball, $b^{\prime}$ and $b^{2}$ at various times $t$, measured in seconds.

| $b^{1}$ |  |
| :--- | :--- |
| $t$ | $b^{1}$ |
| 0 | 50 |
| 1 | 45.1 |
| 1.5 | 39 |
| 2 | 30.4 |
| 2.5 | 19.4 |
| 3 | 5.9 |
| 3.2 | 0 |


| $b^{2}$ |  |
| :--- | :--- |
| $t$ | $h^{1}$ |
| 0 | 50 |
| 1 | 55.1 |
| 1.5 | 54 |
| 2 | 50.4 |
| 2.5 | 44.4 |
| 3 | 35.9 |
| 4.4 | 0 |

a. On this set of axes, plot the points that lie on the trajectory of each ball.

b. Draw and label a smooth curve through each set of points to show the trajectory of each ball.
c. Which ball spent a longer time in the air? $\qquad$ Explain.

$\qquad$


## 〇ロoit Onvestigation

d. Use the graph and approximate the maximum height reached by $b^{2}$ $\qquad$

e. After about how many seconds did $b^{2}$ reach its maximum height? $\qquad$
f. What was the maximum height of $b^{\prime}$ $\qquad$
2. The equations for the parabolas that represent the trajectory of each ball are $h^{1}=4.9 t^{2}+50$ and $h^{2}=-4.9 t^{2}+10 t+50$.
a. What does the coefficient of the linear term in the equation of $h^{2}$ represent?
b. What does the constant in each equation represent? $\qquad$
c. What is the initial velocity of $b^{\prime}$ $\qquad$
3. The formula to find the distance an object travels is $d=v t$, where $d$ is the distance, $v$ is the speed of the object, and $t$ is the time. How many meters from the foot of the wall did each ball fall when it landed?

$$
b^{\prime}:
$$

$\qquad$ $b^{2}$ : $\qquad$
4. Use the formula in (3) and complete the table to calculate the horizontal distance of $b^{2}$ from the wall as it fell. Then, on the axes below, plot the points and sketch the graph of $d=v t$.


| $b^{2}$ |  |
| :--- | :--- |
| $t$ | $d^{2}$ |
| 0 |  |
| 1 |  |
| 1.5 |  |
| 2 |  |
| 2.5 |  |
| 3 |  |
| 4.4 |  |

5 What does the shape of the graph tell you about the motion of the ball?
$\qquad$
$\qquad$


## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations
UNIT 2: Solving Quadratic Equations Using Algebra

## Factoring \& the Zero Product Theorem

As you work through the tutorial, complete the following questions or sentences.

1. The $\qquad$
$\qquad$
$\qquad$ states that if $a$ and $b$ are real numbers and $a b=0$, then $a=0$ or $b=0$.
2. In an equation where the product of two binomials equal to zero, as in $0=(0.4 x+2)(0.4 x-2)$, there are $\qquad$ possible values for the variable x .
3. If a quadratic equation in one variable, $x$, has two roots, then the graph of the equation in two variables has two $\qquad$ -.
4. Once you know the values of the horizontal intercepts, you can find the
$\qquad$
$\qquad$
$\qquad$ and the $\qquad$ of the parabola.
5. If $0=x(x+20)$ then $x=$ $\qquad$ or $x+20=$ $\qquad$
6. Since the graph of the function $y=\pi x^{2}+20 \pi x$ represents he relationship between the width and area of the annulus, it makes sense to select points in quadrant $\qquad$ _.
7. If $x+22=0$ or $x-2=0$, then $x=$ $\qquad$ or $x=$ $\qquad$ .
8. If the factors of a quadratic polynomial in an equation of the form

Key Words:
Zero product theorem Double roots of a quadratic equation

Learning Objectives:

- Recognize that the solutions of a quadratic equation are the $x$-intercepts of the corresponding function.
- Solve a quadratic equation in one variable by factoring the difference of two squares.
- Solve a complete quadratic equation in one variable by factoring.
- Solve a quadratic equation in one variable by factoring a perfect square trinomial. $a x^{2}+b x+c=0$ are the square of a linear binomial, then the corresponding equation has a $\qquad$ . $\qquad$ .

9. The real solutions of the quadratic equation $a x^{2}+b x+c=0$ are the
$\qquad$
$\qquad$ $y=a x^{2}+b x+c$.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I：Course 2

MODULE 3：Quadratic Functions \＆Equations
UNIT 2：Solving Quadratic Equations Using Algebra

## Factoring \＆the Zero Product Theorem

1．The quadratic function $y=0.25 x^{2}-4$ is represented by the parabola shown．
a．Based on the graph，how many x－intercepts does the parabola have？ $\qquad$
b．Solve the quadratic equation algebraically for $y=0$ ．Show your work．

2．Factor the expression $x^{2}+4 x$ ．


$\qquad$

3．Use the equation $y=-3 x^{2}+6 x$ ，to answer the following questions．
a．What are the x－intercepts of the corresponding parabola？ $\qquad$
b．What are the coordinates of the vertex of the parabola？ $\qquad$
c．Using this information，sketch the graph of the parabola．


4．A parabola is defined by the quadratic equation $y=36 x^{2}+24 x+4$ ．
a．Factor $36 x^{2}+24 x+4$ ． $\qquad$
b．What is the value of when $y=0$ ？ $\qquad$
c．How many x－intercepts does the corresponding parabola have？ $\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 2: Solving Quadratic Equations Using Algebra

## The Square Root Method \& Completing the Square

As you work through the tutorial, complete the following questions or sentences.

1. According to the $\qquad$ if $n^{2}=k$, then $n= \pm \sqrt{k}$ for any real number $k$, where $k \geq 0$.
2. The property of opposites states that for any real number $a$,
$\qquad$ and $\qquad$ —.
3. The two number $\frac{6}{5}$ and $-\frac{6}{5}$ are opposites because their $\qquad$ is zero.
4. One way to solve for $x$ in an equation such as $x^{2}+10 x=39$ is to

Key Words:
$\mathrm{Pi}(\pi)$
Volume Isolate
Inverse operation

Learning Objectives:

- Using the properties of equality to rewrite a formula for a particular variable

5. If $(x+5)^{2}=64$, then $x=$ $\qquad$ or $x=$ $\qquad$ .
6. What constant is added to each side of the equation $-1=x^{2}-4 x$ so that the right side is a perfect square trinomial? $\qquad$
7. The equation $x-2= \pm \sqrt{3}$ means that $\qquad$ or $\qquad$ .
8. To find the roots of a quadratic equation of the form $y=a x^{2}+c$ when $y=0$, you can use the $\qquad$
$\qquad$
$\qquad$ .
9. To solve a quadratic equation of the form $a x^{2}+b x+c=0$, where $a=1$ and $b$ and $c$ are rational numbers, use the method of $\qquad$ the $\qquad$ _.
10. If the quadratic expression in the original equation is prime, the real roots will be $\qquad$ -.

$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations
UNIT 2: Solving Quadratic Equations Using Algebra

## The Square Root Method \& Completing the Square

1. How many real solutions are there for he quadratic equation $x^{2}=9$ ?

$\qquad$
2. Find the roots of the following equations using the square root property.
a. $2 x^{2}-18=0$ $\qquad$
b. $15 x^{2}-15=0$. $\qquad$
c. $13 x^{2}-52=0$. $\qquad$
3. What term must be added to the expression $x^{2}+2 b x$ to make it a perfect square trinomial? $\qquad$
4. What term must be added to each of the following expressions so that the result
is a perfect square trinomial?
a. $x^{2}+12 x$ $\qquad$
b. $x^{2}+20 x$ $\qquad$
c. $x^{2}+3 x$ $\qquad$
5. Use the method of completing the square to solve the quadratic equation $x^{2}+4 x-5=0$. Show your work.
6. Use the method of completing the square to solve the quadratic equation $x^{2}-10 x+18=0$. Show your work.
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 2: Solving Quadratic Equations Using Algebra

## The Quadratic Formula

## As you work through the tutorial, complete the following questions or sentences.

1. The quadratic formula states that the solutions of $a x^{2}+b x+c=0$, where $a, b$ and $c$ are real numbers and $a \neq 0$ are:
2. In the quadratic equation $2 x^{2}+8 x-13=0, a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$ —.
3. Express $\sqrt{168}$ in simplest radical form. $\qquad$
4. If the quadratic formula is used to solve an equation for which a corresponding parabola has no $x$-intercepts, then the value of $x$ when $y=0$ is not a $\qquad$ number.
5. The discriminant in the quadratic formula is the expression $\qquad$ .
6. In the quadratic formula, the $\qquad$ is the discriminant.
7. If the discriminant is negative, the equation has $\qquad$ roots.
8. If the discriminant equals zero, the the equation has $\qquad$

Key Words:
The quadratic formula Discriminant

Learning
Objectives:

- Recognize the steps in the proof of the quadratic formula and interpret its meaning.
- Use the quadratic formula to determine that a quadratic equation does not have real roots.
- Use the discriminant to determine the nature of the roots of a quadratic equation in one variable.

9. If the discriminant is positive, the equation has $\qquad$
$\qquad$ roots. root.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I：Course 2

MODULE 3：Quadratic Functions \＆Equations
UNIT 2：Solving Quadratic Equations Using Algebra

## The Quadratic Formula

1．Write the method，property，or theorem used to move from one step to the
 next in each of the following equations．
a．$y^{2}=8 ; y= \pm \sqrt{8}$
b．$y^{2}+6 y=5 ; y^{2}+6 y+9=5+9$ $\qquad$
c．$y(y+3)=0 ; y=0$ or $y+3=0$ $\qquad$
2．In the equation $\mathrm{fg}^{2}+h g+j=0, g$ is the variable，and $f, h$ ，and $j$ are real numbers．Use the quadratic formula and express the value of $g$ in terms of $f, h$ ，and $j$ ． $\qquad$

3．To apply the quadratic formula，an equation must be in the form $a x^{2}+b x+c=0$ ．Use the properties of equality to rewrite each of the following equations in standard form，and identify the values of $a, b$ ，and $c$ ．
a．$x^{2}+12 x=18$
$\qquad$ $b=$ $\qquad$ $c=$ $\qquad$
b． $3 y^{2}+51 x=2 y$
$a=$ $\qquad$ $b=$ $c=$ $\qquad$
c． $2 y-27=8 y^{2}$
$a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$
d．$x+x+2 x^{2}-2=-3 x+x^{2}$ $\qquad$
$a=$ $\qquad$

$$
b=
$$

$\qquad$ $c=$ $\qquad$

4．Use the quadratic formula to solve the equation $5 x^{2}-5 x+1=0$ ． Show your work．

$\qquad$

## MASTERING ALGEBRA I: Course 2

## MODULE 3: Quadratic Functions \& Equations

UNIT 2: Solving Quadratic Equations Using Algebra

## Uniof Assessment

1. This graph represents a rabbit's jump. The variable $x$ represents the horizontal distance in feet of the jump and the $y$-axis represents the height
 in feet of the jump. The equation of the parabola that represents $y=-0.25 x^{2}+0.5 x$.
a. How many solutions does $x$ have when $y=0$ ?
$\qquad$ Why would this make sense in describing a rabbit's jump? $\qquad$
b. What is $x$ when $y=0$ ? $\qquad$
c. How far did the rabbit jump? $\qquad$

d. How high did the rabbit jump? $\qquad$
2. Solve the quadratic equation $0=m^{2}-81$. Show your work.


Distance Jumped (ft)
3. Solve the quadratic equation $0=s(s-99)$. Show your work.
4. To use the method of completing the square to solve a quadratic equation of the form $x^{2}+b x=c$, what term must be added to both sides of the equation? $\qquad$
5. Use the method of completing the square to solve the quadratic equation $x^{2}+18 x-19=0$. Show your work.
$\qquad$
$\qquad$

6. Use the quadratic formula to solve the equation $0=x^{2}+7 x+5$. Show your work.

## Unoiช Assessment

7. What does the discriminant tell you about the solutions of a quadratic equation?
$\qquad$
$\qquad$
8. Calculate the discriminant, then describe the nature of the roots for each of the following quadratic equations.

| Quadratic equation | Discriminant | Nature of roots |
| :--- | :--- | :--- |
| a. $5 x^{2}+6 x+5=0$ |  |  |
| b. $6 x^{2}+6 x+7=0$ |  |  |
| c. $2 x^{2}+8 x+2=0$ |  |  |
| d. $8 x^{2}+3 x-4=0$ |  |  |

9. Which of the following equations correspond to parabolas that have exactly one x-intercept.
a. $y=5 x^{2}+10 x+5$
b. $y=0.25 x^{2}+2 x+4$
c. $y=4 x^{2}+3 x+4$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 3: Quadratic Functions \& Equations UNIT 2: Solving Quadratic Equations Using Algebra

## Investigating Parabolic Motion

1. Gravity affects all falling objects. The equation $h=\frac{1}{2} g r^{2}+c$ represents the trajectory of a falling object, where the variables $t$ and $h$ represent
 the time measured in seconds, and the distance fallen, measured in meters. The constants in the equation are $g$ and $c$, where $g$ is the acceleration constant, $-9.8 \mathrm{~m} / \mathrm{s}^{2}$, and c is the height from which an object is dropped.
a. Suppose a boulder falls from the top of a cliff that is 60 feet high.

What equation can you write to represent the trajectory of the boulder in terms of $t$ and $h$ ?
$h=$ $\qquad$ $t^{2}+$ $\qquad$
b. Use the equation in (a) and complete this table. Round values of $h$ to the nearest whole number.

| $t$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $h$ |  |  |  |  |  |

c. Between what two values of $t$ will the boulder strike the ground?


Explain. $\qquad$
$\qquad$
d. On the axes above, plot the points from the table, and sketch the parabola that represents the trajectory of the boulder.
e. What was the maximum height in meters of the boulder above the ground? $\qquad$
f. Calculate the value of $t$ to the nearest tenth when the boulder struck the ground. Show your work below.
$\qquad$


## 〇๓it Onvestigation

2. Two players, $A$ and $B$, are trying out for a position on the softball team. Whoever throws the ball farther will make the team. The equation that represents the trajectory of a ball thrown by player A is $\mathrm{y}=0.04 x^{2}+x+2$. The equation that represents the trajectory of $a$ ball thrown by player $B$ is $y=0.05 x^{2}+1.18 x+2$. In each equation, $x$ represents the horizontal distance in feet between the ball and a player, and $y$ represents the height in feet of the ball above each player's shoulder.
a. What is the initial height of the ball thrown by each player?.

Player A: $\qquad$
Player B: $\qquad$
b. Use the two equations above and for each value of $x$, calculate the values, to the nearest tenth, for $y_{A}$ and $у$.

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y A$ |  |  |  |  |  |  |  |
| $y B$ |  |  |  |  |  |  |  |


c. Plot the points from the table and sketch the trajectory of each ball.
d. Use the graph and tell which player threw the ball higher.
e. Use the graph and approximate the horizontal distance that each player threw the ball. Player A: $\qquad$ ff. Player B: $\qquad$ ft.
f. Use the quadratic formula and in the space below, calculate the distance, in feet, that the farther ball traveled. Round your answer to the nearest tenth.
g. Which player made the team? Player $\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 4: Algebraic Expressions \& Functions
UNIT 1: Radical Equations \& Functions


## Solving Radical Equations

As you work through the tutorial, complete the following questions or sentences.

$$
1 . A
$$

$\qquad$
$\qquad$ is an equation in which a variable is under a radical sing.
2. If $a$ and $b$ are real numbers and $a=b$, then $\qquad$ -.
3. The $\qquad$
$\qquad$ of a number can be expressed using an exponent of $\qquad$ -
4. Another way to solve a radical equation is to rewrite the expression using the exponent $\frac{1}{2}$, and then to $\qquad$ both sides of the equation.
5. Explain how graphing the system of equations $y=\sqrt{x}$ and $y=2$ can be used to check the solution of the radical equation $\sqrt{x}=2$. $\qquad$

Key Words:
Radical equation Extraneous root

## Learning

 Objectives:- Recognize and solve a simple radical equation.
- Determine if a radical equation has a real solution.
- Solve radical equations algebraically.
- Determine if a radical equation has an extraneous solution.

6. An $\qquad$
$\qquad$ is a solution of the square of a radical equation that is not a solution of the original radical equation.
7. Rewrite as an exponential equation. $\sqrt{x}=$ $\qquad$ if $x \geq 0$.
8. True or false: A radical equation may have no solution, one solution, or two solutions. $\qquad$
9. Always check the solutions of the square of a radical equation. However, do NOT include $\qquad$ in the solution set of the original radical equation.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 4: Algebraic Expressions \& Functions
UNIT 1: Radical Equations \& Functions

## Solving Radical Equations

1. Solve the equation $\sqrt{x}=5$ for $x$.

2. Would it make sense to attempt to solve the radical equation $\qquad$
$\sqrt{m}=-49$ ? $\qquad$ Why or Why not? $\qquad$
$\qquad$
3. Solve and check the equation $\sqrt{r}-5-8=0$ for $r$. Show all work.
4. Consider the functions
$y=\sqrt{x}+2$ and $y=x$.
a. Create a scale, and Plot the graphs of the two functions on the same set of axes.
According to the graph, how many solutions do you expect for the radical equation
$\sqrt{x}+2=x$ ? $\qquad$

b. Solve the radical equation $\sqrt{x}+2=x$ and check your work.
c. What is the solution of the equation $\sqrt{x+2}=x$ ? $\qquad$
d. What is the extraneous solution if any? $\qquad$

$\qquad$

## MASTERING ALGEBRA I: Course 2

 MODULE 4: Algebraic Expressions \& Functions UNIT 1: Radical Equations \& Functions
## The Inverse of the Square Root Functon



As you work through the tutorial, complete the following questions or sentences.

1. A $\qquad$ function is a function in which each second coordinate is paired with one and only one first coordinate.
2. The $\qquad$ of a one-to-one function obtained by interchanging the two variables in the one-to-one function.
3. When you $\qquad$ the variables of a one-to-one function, the relation you get is a $\qquad$ , too, which is the $\qquad$ of the first function.
4. The inverse of $f(x)$ is denoted by the notation $\qquad$ and is read as " $f$ inverse of $x$."
5. If $f(x)=\sqrt{x}$, then the domain of $f^{-1}(x)$ is the $\qquad$ of $f(x)$.
6. If $f(x)=\sqrt{x}$, then the range of $f^{-1}(x)$ is the $\qquad$ of $f(x)$.

Key Words:
Inverse of a
function
One-to-one
function

Learning

## Objectives:

- Graph the inverse of the square root function, and identify its equation.
- Calculate the equation of the line of symmetry between the square root function and its inverse.
- Examine the inverse of a parabolic function with a restricted domain.

9. The relationship between the points $(x, y)$ on the graph of the inverse of $y=x^{2}$ is $\qquad$ .
10. You can investigate the inverse of a function that is not a one-to-one function by restricting its $\qquad$ .
$\qquad$
$\qquad$

## MASTERING ALGEBRA I：Course 2

MODULE 4：Algebraic Expressions \＆Functions UNIT 1：Radical Equations \＆Functions

## The Inverse of the Square Root Functon

1．Is the function shown in the graph a one－to－one function？ $\qquad$
Why or why not？ $\qquad$
$\qquad$
$\qquad$
$\qquad$

2．a．Create a scale and a graph the
 function $f(x)=\sqrt{x}-1$ ．

b．What is the domain of $f(x)$ ？ $\qquad$
c．What is the range of $f(x)$ ？ $\qquad$
d．What is the equation of $f^{-1}(x)$ ？ $\qquad$
e．What is the domain of $f^{-1}(x)$ ？ $\qquad$
f．Graph $f^{-1}(x)$ on the same set of axes above．
g．Graph the line of symmetry between the graph of $f(x)$ and the graph of $f^{-1}(x)$ ，and write its equation． $\qquad$
$\qquad$
$\qquad$

|  |  |  |  |
| :---: | :---: | :---: | :---: |
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|  |  |  | \#: Radica |

## Unois Assessment

Solve and check the equations below. Show all your work. State all real solutions, and which, if any, roots are extraneous. If an equation has no solution, write "no solution."


1. $\sqrt{a=12}$
2. $\sqrt{k+2}=0$
3. $\sqrt{d-15}=0$
4. $z=\sqrt{z+7}+5$
5. Graph $y=\sqrt{\overline{x+3}}$ and $y=x+1$ on the set of axes below. Based on your graphs, how many solutions are there to the equation
$\sqrt{x+3}=x+1$ ? $\qquad$ What is it? $\qquad$

$\qquad$

6. Solve and check the equation $\sqrt{x}-1-1=\frac{x}{5}$. Show your work.

7. Is the function shown in this graph a one-to-one function? $\qquad$ Why or why not? $\qquad$
$\qquad$
$\qquad$

8. Consider the function $f(x)=\sqrt{x+2}$
a. What is the domain of $f(x)$ ? $\qquad$
b. What is the range? $\qquad$
c. What is the equation of $f^{-1}(x)$ ? $\qquad$
d. What is the domain of $f^{-1}(x)$ ? $\qquad$
e. What is the range of $f^{-1}(x)$ ? $\qquad$

f. Graph $f(x)$ and $f^{-1}(x)$ on the same set of axes. Show the line of symmetry between the graphs.
$\qquad$


## MASTERING ALGEBRA I: Course 2

MODULE 4: Algebraic Expressions \& Functions UNIT 1: Radical Equations \& Functions

## Unio Assessment

## Investigating Gravity

The properties of radical equations can be used to discover how certain objects are affected by gravity. Suppose a large rock is dropped from a 256-foot high cliff.
The equation $t=\sqrt{16-}\left(\frac{1}{16}\right) d$ can be used to determine how long it will take the rock to fall a given distance. In the equation, the variable $t$ represents the amount of time in seconds the object has been falling, and the variable $d$ represents the height of the object in feet above the ground.

1. What is the distance between the rock and the ground after 2 seconds?
$\qquad$ Show your work.
2. Exactly 3 seconds after the rock is dropped, a bird flies across the vertical path of the falling rock at a height of 120 feet above the ground. Is the bird in danger? $\qquad$ Why or why not?

$\qquad$
3. Draw the graph of $\left.t=\sqrt{16-\left(\frac{1}{16}\right.}\right) d$

## Unit 0muestigaetion

4. Since $d$ represents the height above the ground in feet, it has restricted values. What set of values is appropriate for $d$ ? $\qquad$
$\qquad$
5. At what time, $t$, is the rock halfway to the ground? Round your answer to the nearest tenth. $\qquad$ Show your work.
6. At what time, $t$, does the rock hit the ground? $\qquad$ Show your work.
7. What is the range of the function for values of $d$ from 0 to 4 ? $\qquad$ Explain your answer.
8. Is the function a one-to-one function? $\qquad$ Explain.

$\qquad$

## MASTERING ALGEBRA I: Course 2

 MODULE 4: Algebraic Expressions \& Functions UNIT 2: Rational Expressions, Equations \& Functions
## Rational Functions

As you work through the tutorial, complete the following questions or sentences.

1. A $\qquad$ is a fraction whose numerator and denominator are polynomials, and the degree of the polynomial in the denominator is at least $\qquad$ _.
2. If $a, b, c$, and $d$ are real numbers, and $c$ and $d$ are not equal to zero, $\frac{a \times b}{c \times d}=$ $\qquad$ .
3. If $a$ is a non-zero real number, $\qquad$ is undefined.
4. Any value of a variable that results in a zero denominator is an
$\qquad$
$\qquad$ in a rational expression.
5. Does finding the quotient of two rational expressions follow the same rules as dividing fractions? $\qquad$ Explain your answer.
6. If $a, b$, and $c$ are real numbers and $b \neq 0$, then $\frac{a \times c}{b \times b}=$ $\qquad$
7. If $a, b, c$, and $d$ are real numbers, and $b$ and $d$ are not equal to zero, then $\frac{a \times c}{b \times d}=$ $\qquad$
8. To find the excluded value of $x$ in a rational expression, set the
$\qquad$ of the expression equal to 0 and solve for $\qquad$ .

Key Words:
Ratio
Rational expression
Excluded values

Learning
Objectives:

- Identify value(s) of x for which a rational expression is undefined.
- Reduce a rational expression by removing common monomial factors.
- Find the product or quotient of two rational expressions and express it in simplest form.
- Identify the least common denominator of two algebraic fractions.
- Find the sum or difference of two rational expressions and express it in simplest form.

9. If $a, b, c$, and $d$ are real numbers, and $b, c$, and $d \neq 0$, then $\frac{a}{c} \div \frac{b}{d}=\frac{a}{c} \times$ $\qquad$ _.
$\qquad$
$\qquad$

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
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## MASTERING ALGEBRA I: Course 2

MODULE 4: Algebraic Expressions \& Functions
UNIT 2: Rational Expressions, Equations \& Functions
Rational Functions


1. Find the excluded value(s) for each rational expression.
a. $4 a+4$
$a+2$
$\qquad$

b. $\frac{n^{2}+3 n+2}{6 n}$ $\qquad$
C. $\frac{t-5}{3 t+3}$ $\qquad$
d. $\frac{x^{3}-x^{2}+5 x+22}{x^{2}-4}$ $\qquad$
2. Determine the excluded value(s) of $x$ for each rational expression and then simplify the expression.
a. $\frac{b+2}{b^{2}} \times \frac{5 b}{b^{2}-4}, b \neq$
b. $\frac{c^{2}-9}{5 c} \div \frac{c-3}{20}, c \neq$
$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

 MODULE 4: Algebraic Expressions \& Functions
## UNIT 2: Rational Expressions, Equations \& Functions

## Rational Functions

As you work through the tutorial, complete the following questions or sentences.

1. A $\qquad$
$\qquad$ is an equation that contains one or more rational expressions.
2. A plane curve represented by an equation such as $y=\frac{1}{x}$ is called a
$\qquad$ _.
3. If zero is not included in the domain or the range of a function, then the graph of the function cannot intersect the $\qquad$ or the
$\qquad$
$\qquad$ .
4. Use symbols to complete these expressions for the function $y=\frac{1}{x}$. As $x \rightarrow 0^{+}, y$ $\qquad$ As $x \rightarrow 0^{-}, y$ $\qquad$

As $x \rightarrow+\infty, y$ $\qquad$ As $x \rightarrow-\infty, y$ $\qquad$
5. A line that a hyperbola approaches but never intersects is called an
$\qquad$ of the hyperbola.
6. The domain of $y=\frac{1}{x}$ does not include 0 , but does include numbers on both sides of 0 . Therefore the graph is a $\qquad$ function.

Key Words:
Rational function Hyperbola Asymptote Continuous function Discontinuous function

Learning Objectives:

- Graph the function $f(x)=\frac{1}{x}$..
- Identify the domain and range of $f(x)=\frac{1}{x}$.
- Identify the equations of the asymptotes and the inverse of $f(x)=\frac{1}{x}$.
- Examine the effects of parameters $a$ and $b$ on the graph of $f(x)=\frac{a}{x-b}$

7. For any rational function $y=\frac{a}{x-b}, b$ represents the $\qquad$
$\qquad$ of the rational expression and $x=b$ is the equation of the
$\qquad$
$\qquad$ —.
8. For any rational function $y=\frac{a}{x-b}$, changing the value of $b$ causes the hyperbola to shift $\qquad$ -.
9. For any rational function $y=\frac{a}{x-b}$, the parameter $a$ shifts the hyperbola toward or away from the $\qquad$ _.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I：Course 2

MODULE 4：Algebraic Expressions \＆Functions
UNIT 2：Rational Expressions，Equations \＆Functions

## Rational Functions

1．Use the symbols＜，＞，or $=$ to complete each statement for the corresponding graph，which represents the equation $y=\frac{a}{x-b}$ ．

a．a $\qquad$ 0

b．$a$ $\qquad$

c．$b$ $\qquad$ 0

d．$b$ $\qquad$ 0


2．Each tick mark in the axes represents 1 ．What is the equation of the vertical asymptote for this hyperbola？


3．Sketch the graph of the hyperbola that represents each equation．Include the vertical asymptote of each hyperbola and equation．
a．$y=\frac{4}{x-2}$
b．$y=\frac{4}{x+2}$


$\qquad$

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## MASTERING ALGEBRA I: Course 2

MODULE 4: Algebraic Expressions \& Functions UNIT 2: Rational Expressions, Equations \& Functions

## Rational Equations

As you work through the tutorial, complete the following questions or sentences.

1. To solve $\frac{3}{x}=\frac{1}{x-2}$ for $x$, first find the excluded values, then find a
$\qquad$
$\qquad$ .
2. What is the solution of the rational expression $\frac{3}{x}=\frac{1}{x-2}$ ? $\qquad$
3. The graphs of $y=\frac{3}{x}$ and $y=\frac{1}{x-2}$ intersect at a point whose $x$-coordinate is $\qquad$ ـ.
4. The formula for work is $\qquad$ $\times$ $\qquad$ $=$ $\qquad$ .
5. What is the lowest common denominator of $\frac{5}{7}$ and $\frac{5}{x}$ ? $\qquad$
6. If $d=r t$, which equation expresses time in terms of distance and rate?
7. An $\qquad$
$\qquad$ is a solution of a modified rational equation that is not a solution of the original rational equation.
8. Before solving a rational equation, identify the $\qquad$
$\qquad$ of the variable.
9. One way to solve a $\qquad$
$\qquad$ is to multiply each term in the equation by a common nonzero denominator to reduce like factors.

Key Words:
Rational equation Lowest common denominator Extraneous solution

## Learning

## Objectives:

- Solve a rational equation by multiplying by the LCD.
- Analyze and solve a work problem.
- Analyze and solve a uniform motion problem.
- Determine if a solution of a rational equation is extraneous.
$\qquad$
$\qquad$
$\qquad$


## MASTERING ALGEBRA I：Course 2

MODULE 4：Algebraic Expressions \＆Functions UNIT 2：Rational Expressions，Equations \＆Functions

## Rational Equations

1．Find the lowest common denominator for the fractions in each equation．

a．$\frac{5}{2 k}=\frac{3}{k+4}$ $\qquad$
b．$\frac{2+w}{12 w}=\frac{2}{w-3}$ $\qquad$
c．$\frac{4 p}{p 2-5 p+6}=\frac{14}{p-3}$ $\qquad$
2．Solve for $p$ in this rational equation． $\qquad$

$$
\frac{4 p}{p 2-5 p+6}=\frac{14}{p-3}
$$

3．Solve for $b$ in this rational equation．

$$
\frac{4 p}{p 2-5 p+6}=\frac{14}{p-3}
$$

4．One person can mow a lawn in two hours．Two people，working together，can mow the same lawn in 1 hour and 15 minutes．How many hours would it take the second person to mow the lawn alone？

5．A local bus leaves the station at 7：30 A．M．An express bus leaves the station 20 minutes later，travels the same route，and arrives at the final stop 10 minutes before the local bus．
a．Use the formula $r=d / t$ and write an expression in terms of $r$ and $t$ to find the rate of the express bus．
b．If the local bus averages 20 miles per hour，and the express bus averages 50 miles per hour，what time does the express bus arrive at the final stop？ $\qquad$

6．A student can bicycle to school in 10 minutes less time than it takes to walk the same distance．The student bicycles at a rate four times faster than the walking rate．How long does it take the student to ride a bike to school？ $\qquad$

$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 4: Algebraic Expressions \& Functions UNIT 2: Rational Expressions, Equations \& Functions

## Uniot Assessment

1. Find the excluded value(s) for each rational expression.

a. $\frac{6 h-2}{2 h^{2}}, h \neq$ $\qquad$
b. $\frac{4 g}{3+g}, g \not \approx$ $\qquad$
c. $\frac{d+2}{d-2}, d \neq$ $\qquad$
2. Simplify each expression.
a. $\frac{3 w}{w-2} \times \frac{2 w+6}{3 w}$
b. $\frac{2 z}{z-1}+\frac{3}{z+1}$ $\qquad$
c. $\frac{4-u}{12 v} \div \frac{3}{v}$
d. $\frac{8}{y+2}-\frac{y+3}{6}$ $\qquad$
3. Complete the table of values for the function $f(x)=\left(\frac{2}{x+3}\right)$.

| $\mathbf{x}$ | $f(\mathbf{x})$ |
| :---: | :---: |
| -4 |  |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

$\qquad$

## Unoi̛ Assessment

4. Sketch the graph of each function. Indicate each asymptote.
a. $f(x)=\frac{2}{x-6}$
b. $f(x)=\frac{1}{x+1}$


5. A house painter can paint a room in 6 hours. If an assistant helps, the two can paint the room in 4 hours. How long would it take the assistant working alone to paint the room? $\qquad$

6. A runner leaves the gym and sets out on a training run at a rate of 6 miles per hour.
a. After half an hour, the runner increases his rate to 8 miles per hour and runs for another 15 minutes. What is the total distance the runner travels?
b. Suppose the runner maintains a steady rate of 6 miles per hour. The runner's coach leaves the gym on a bicycle 30 minutes after the runner leaves and catches up with the runner 15 minutes later. At what rate does the coach travel?
c. If the coach leaves the gym one hour after the runner does and rides at the same rate as in part (b), how long will it take the coach to catch up with the runner?
$\qquad$

$\qquad$


## MASTERING ALGEBRA I: Course 2

 MODULE 4: Algebraic Expressions \& Functions UNIT 2: Rational Expressions, Equations \& Functions
## บ๓i̊ Omveతtigeగfon

## Investigating Waves



The frequency of a wave-such as a sound wave, electromagnetic wave, or mechanical wave-is represented by the equation $f=\frac{l}{f}$, where $f$ is the frequency of the wave, and $t$ is the time required for one complete vibration of the wave, which is called the period.

1. Graph the function $f=\frac{1}{f}$ and answer the following questions.

2. As the frequency of the wave increases, what happens to the period of the wave? $\qquad$
3. As the period of the wave increases, what happens to the frequency of the wave? $\qquad$
4. When a child swings on a swing set, the motion of the swing can be modeled as wave motion. Sketch the graph of $f=\frac{1}{f}$ to find the frequency of the wave if they swing makes one complete back-and-forth motion in 2 seconds. $\qquad$
5. Use the graph to find the period of the wave if an amusement-park swing ride makes four complete back-and-forth motions every second.
$\qquad$
$\qquad$

6. The speed of a wave is represent by $v=\frac{\lambda}{t}$ where $v$ is the speed of the wave, and $\lambda$ is a constant that represents the wavelength of the wave. Graph the function for a wave with a wavelength of 2.0 m .

7. Compare this graph with the graph of $f=\frac{1}{t}$. Then sketch a graph of a wave with a wavelength of 2.5 m .

8. According to the graphs, how does increasing the wavelength of a wave affect the time required for one complete vibration?
$\qquad$

$\qquad$

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## MASTERING ALGEBRA I: Course 2

## MODULE 5: Describing Data

## UNIT 1: Graphical Displays

## Stem-\&-Leaf Plots \& Box Plots

$$
\begin{aligned}
& \text { Stఇdent } \\
& \text { Logbook }
\end{aligned}
$$

## As you work through the tutorial, complete the following questions or sentences.

1. Statistics is the mathematical study of the $\qquad$
$\qquad$ —, and $\qquad$ of data.
2. A $\qquad$ is a type of bar graph that represents
$\qquad$ in a data set.
3. A graphical display made up of the numbers in a distribution is a
$\qquad$ -and- $\qquad$ plot.
4. The range of data is the $\qquad$ between the maximum and minimum values in a data set.
5. A distribution of data that is asymmetrical is called $\qquad$ .
6. The $\qquad$ is the middle number in a set of ordered data.
7. Quartiles are the three numbers that divide a set of data into
$\qquad$ equal parts.
8. The median of the data is the $\qquad$
$\qquad$ .
9. A $\qquad$
 $\qquad$ is a graphical display that divides a set of data into quartiles.
10. The interquartile range is the difference between the $\qquad$ and
$\qquad$ quartiles of a distribution.
11.An $\qquad$ is an extreme or uncharacteristic value in a data set.
$\underbrace{\text { ( }}$
$\qquad$
$\qquad$
$\square$

## MASTERING ALGEBRA I: Course 2

MODULE 5: Describing Data
UNIT 1: Graphical Displays

## Stem-\&-Leaf Plots \& Box Plots

## Use the data table on the right to answer Questions 1-8.

| City | Percent |
| :--- | :---: |
| Chicago, IL | 29.7 |
| Dallas, TX | 6.7 |
| Detroit, MI | 10.7 |
| Houston, TX | 6.5 |
| Los Angeles, CA | 10.5 |
| New York, NY | 53.4 |
| Philadelphia, PA | 28.7 |
| Phoenix, AZ | 3.3 |
| San Antonio, TX | 4.9 |
| San Diego, CA | 4.2 |

1. The data in the table show the percentage of workers who used public transportation in the ten largest U.S. cities in 1990. Use the data to create a stem-and-leaf plot. Be sure to include a key.

$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 5: Describing Data
UNIT 1: Graphical Displays

## Scatter Plots \& Linear Best-Fit Graphs

## As you work through the tutorial, complete the following statements and questions.

1. A scatter plot can be used to represent all of the data in $\qquad$ dimensions.
2. A line that best represents the relationship between the points in a scatter plot is called a $\qquad$ line.
3. A best-fit line drawn on a scatter plot that uses a three-point summary based on the medians of the variables in three groups of data is called a
$\qquad$ - $\qquad$ line.
4. Does the line $M_{1} M_{3}$ represent all of the data? $\qquad$ Why or why not?
5. The best-fit median-median line is $\qquad$ to both the line $M_{1} M_{3}$ and the line through $M_{2}$, and must lie $\qquad$ of the distance from $M_{1} M_{3}$ and $\qquad$ of the distance from the line through $M_{2}$.

Key Words:
Data
Median
Outlier
Scatter plot
Best-fit line
Median-median line

## Learning

Objectives:

- Approximate a best-fit line through a set of points in a scatter plot.
- Use a best-fit line to predict a future value.
- Compare actual and predicted values using a best-fit line.

6. A best-fit line can suggest only a general $\qquad$ in the relationship between two variables.
7. The equation of a best-fit line for a set of data can be used to make
$\qquad$ —.
$\qquad$
$\qquad$

## MASTERING ALGEBRA I: Course 2

## MODULE 5: Describing Data

UNIT 1: Graphical Displays

## Scatter Plots \&

## Linear Best-Fit Graphs

1. The table on the right lists the winning times for men's 100-meter freestyle swim during the Olympic Games from 1924 to 1988. Graph the relationship as a scatter plot. Be sure to label the axes and put the independent variable on the vertical axis.

2. Divide the data on your graph into three equal groups. Find the median coordinates

| Year | Winning Time(s) <br> (seconds) |
| :--- | :---: |
| 1924 | 59.0 |
| 1928 | 58.6 |
| 1932 | 58.2 |
| 1936 | 57.6 |
| 1948 | 57.3 |
| 1952 | 57.4 |
| 1956 | 55.4 |
| 1960 | 55.2 |
| 1964 | 53.4 |
| 1968 | 52.2 |
| 1972 | 51.22 |
| 1976 | 49.99 |
| 1980 | 50.40 |
| 1984 | 49.80 |
| 1988 | 48.63 | for each group and label these points $M_{1} M_{2}$ and $M_{3}$.

3. Find the equation for each of the following lines.
a. The one through point $M_{1}$ and $M_{3}$ $\qquad$
b. The line through $M_{2}$ that is parallel to $M_{1} M_{3}$
c. The median-median best-fit line $\qquad$
4. Use the equation for the median-median line to predict the winning time in 1992.
$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 5: Describing Data
UNIT 1: Graphical Displays

## Unoi̛ Assessment

1. The data table on the right shows the average speeds in miles per hour of several animals. Use the data to create a stem-and-leaf plot. Be sure to include a key.
Key: $\qquad$

| Animal | Speed (in mph) |
| :--- | :---: |
| Cheetah | 70 |
| Coyote | 43 |
| Elephant | 25 |
| Elk | 45 |
| Giraffe | 32 |
| Gray fox | 42 |
| Grizzly bear | 30 |
| Lion | 50 |
| Reindeer | 32 |
| Zebra | 40 |

a. What is the mean of the data? $\qquad$
b. What is the outlier? $\qquad$
c. What is the median of these data? $\qquad$
d. Does the mean represent a valid measure of central tendency in this data set? $\qquad$ Explain your answer.
2. a. Determine the lower, middle, and upper quartiles for the data in the table in Question 1. Then use the information to create a box plot of the data.
Q1 = $\qquad$
Q2 = $\qquad$
Q3 $=$ $\qquad$

b. What is the interquartile range of the data?
c. What does the interquartile range tell you abut the data?

$\qquad$
$\qquad$

## Unoił Assessment

3. State whether the following types of graphical display are used to display one-dimensional or two-dimensional data.
a. Stem-and-leaf plot $\qquad$
b. Box plot $\qquad$
c. Scatter plot $\qquad$
4. The table on the right shows the life expectancy in years for women from 1930 to 1990. Without graphing the data, answer the following questions.
a. Divide the data into three roughly equal groups of four, five, and four values, and find the coordinates of $M_{1} M_{2}$ and $M_{3}$.
$\qquad$ , $\qquad$ , $\qquad$ .
b. Calculate the slope of the median-median best-fit line. $\qquad$
c. What is the equation for the median median best-fit line? $\qquad$
d. Use the median-median line to predict the - life expectancy for women in 1995.

| Year | Life Expectancy for <br> Women (in years) |
| :---: | :---: |
| 1930 | 61.6 |
| 1935 | 63.9 |
| 1940 | 65.2 |
| 1945 | 67.9 |
| 1950 | 71.1 |
| 1955 | 72.8 |
| 1960 | 73.1 |
| 1965 | 73.7 |
| 1970 | 74.8 |
| 1975 | 76.6 |
| 1980 | 77.4 |
| 1985 | 78.2 |
| 1990 | 78.8 |


$\qquad$

## MASTERING ALGEBRA I: Course 2

MODULE 5: Describing Data UNIT 1: Graphical Displays

## Investigating National Parks

1. The table on the right lists the national parks in Arizona, Colorado, Utah, and Wyoming, along with the number of years that each park has been a national park. Suppose you want to know the range of years that the middle $50 \%$ of the parks have been national parks. Identify and create the type of data distribution that most clearly shows this range. Then calculate this middle range and identify any outliers in the data. Show your graph and work below.

| National <br> park | Years <br> Established |
| :--- | :---: |
| Arches | 28 |
| Bryce Canyon | 75 |
| Canyonlands | 35 |
| Capitol Reef | 28 |
| Grand Canyon | 80 |
| Grand Teton | 70 |
| Mesa Verde | 93 |
| Petrified Forest | 37 |
| Rocky Mountain | 84 |
| Saguaro | 5 |
| Yellowstone | 127 |
| Zion | 80 |

$\qquad$
$\qquad$


## Unỉ Onvestigation

2. Using the table in Question 1, identify and create the type of data distribution that most clearly shows the number of years since each
 national park was established. Identify the maximum and minimum years, and explain whether or not the mean of the data represents the best measure of central tendency in the data set. Show your work below.
3. The table on the right lists the total number of visitors to all national parks during a 45 -year period. Create a graph that shows if there is a general trend in the data. Using what you have learned about the median-median best-fit line, predict the number of visitors for the year 2000. Explain whether your prediction is reasonable. Show your work below.

| Year | Number of <br> Visitors |
| :--- | :--- |
| 1990 | 57,700 |
| 1985 | 50,000 |
| 1980 | 60,200 |
| 1975 | 58,800 |
| 1970 | 45,879 |
| 1965 | 36,566 |
| 1960 | 26,630 |
| 1955 | 18,830 |
| 1950 | 13,919 |
| 1945 | 4,538 |



### 1.1 Rational \& Irrational Numbers

## Defining Real Numbers

Student Logbook

1. integers; 0
2. terminates; repeats
3. two; number
4. ratio; integers
5. terminating; repeating
6. rational; irrational
7. real
8. root
9. radical

## Your Turn

1. The following answers are samples
a. $-\frac{12}{2}-\frac{18}{3} ;-\frac{30}{5}$
b. $\frac{2}{10} ; \frac{-3}{-15} ; \frac{4}{20}$
c. $\frac{16}{6} ; \frac{-24}{9} ;-\frac{32}{12}$
d. $\frac{9}{4} ; \frac{-9}{-4} ; \frac{18}{8}$
2. a. 0.375
b. 0.222 ...
c. 3.5
d. 0.714285 ..
3. 1.222343
4. Sample answers: $\sqrt{7}, 5.1682032412 \ldots$
5. a. 2.646
b. 5.916
c. 4.690
d. 3.742


## Working with Radicals

## Student Logbook

1.3.14
2. $d$, the depth of the water
3. radicand
4. $1,4,9,16,25$
5. $\sqrt{a^{2}}=a$
6. non-negative square root
7. $\sqrt{a \times b}$
8. $5 \sqrt{10}$
9. $\frac{\sqrt{a}}{\sqrt{b}}$
10. rationalize; denominator; rational
11. simplest; like

## Your Turn

1. a. 9; 9
b. $625 ; 25$
c. $144 . ; 12$
2. $121,144,169,196,225$
3. a. $4 \sqrt{10}$
b. $30 \sqrt{3}$ c. $-\frac{1}{2} \sqrt{10}$
4. $-96 \sqrt{7}$
5. $\frac{\pi}{6}$
6. a. $\frac{\sqrt{6}}{6}$
b. $\frac{\sqrt{33}}{11}$
c. $\frac{\sqrt{14}}{7}$
7. a. $\frac{16}{5}$
b. $5 \sqrt{5}$
C. $\frac{2 \sqrt{3}}{3}$
8. $16 \sqrt[5]{2}$
9. $2 \sqrt{3}$ seconds

## The Square Root function

## Student Logbook

1. $\sqrt{x}$
2. Sample answer: The slope formula can be used to show that slopes between any two consecutive points are not equal.
3. Because for each first coordinate there is one and only one second coordinate.
4. interpolate; domain
5. extrapolate; observed
6. non-negative real numbers
7. non-negative real numbers
8. parameter
9. steepness; quadrant

## Your Turn

1. 

| $x$ | 0 | 1.3 | 2.0 | 2.7 | 3.4 | 4.8 | 5.2 | 5.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sqrt{x}$ | 0 | 1.14 | 1.41 | 1.64 | 1.84 | 2.19 | 2.28 | 2.43 |

2. 


3.

4. a. 3
b. 4
c. 2
d. 1
e. 5

## Unit Assessment

1. a. rational, terminating decimal
b. rational, can be written as $\frac{25}{14}$
c. irrational, nonterminating and nonrepeating decimal ( $\pi$ is irrational)
d. rational, repeating decimal
e. rational, can be written as -3
f. irrational, nonterminating and nonrepeating decimal (2.8284 ...)
g. rational, repeating decimal
h. irrational, nonterminating and nonrepeating decimal
2. sometimes
3. always
4. Irrational; by the Pythagorean theorem, the length of the hypotenuse is given by the square root of $4^{2} \times 5^{2}$, or the square root of 41. No rational number can be squared to equal 41 , so the length of the hypotenuse is given by an irrational number.
5. $b$ and d
6. 


7. $c$ and d
8. a. $5 \sqrt{3}$
b. 0.06
c. $5 \sqrt{17}$
d. $14 \sqrt{7}$
e. $4 \sqrt{3}$
f. $2 \sqrt{3}$
g. $3 \sqrt{5}$
h. $30 \sqrt{3}$ i. $\frac{3 \sqrt{5}}{5}$
i. $\sqrt{5}$
9. c
10.a. 4
b. 3
c. 2
d. 5
e. 1

## Unit Investigation

1. 


2. Answer will vary. ( 0,0 ) , $5,000,86.26 \ldots$...) (15,000, 149.41...), (25,000, $192.89 \ldots$...), (35,000, 228.24 ...)
3. The distance to the horizon when the plane is on the ground
4. about 210 miles
5. about 7,000 feet
6. about 27,000 feet
7. The distance to the horizon decreases from about 230 miles to 190 miles.

### 2.1 Polynomial Arithmetic

## Working with Powers

## Student Logbook

1. exponent; base
2. 1
3. $\frac{1}{\mathrm{a}}$
4. reciprocal; opposite
5. base
6. $a^{r+s}$; integers
7. $a^{r-s}$; integers
8. $a^{r \times s}$; integers
9. $a^{n} \times b^{n}$; integer
10. $\frac{a^{n}}{b^{n}}$; integer

## Your Turn

1.a. $3^{2}$
b. $3^{-2}$ c. $3^{4}$ d. $3^{2}$
2. 1
3. a. $b^{5}$
b. $-3 c^{2}$
c. $25^{6}$
d. $\frac{1}{3}$ or $3^{-1}$
e. $32 x^{15} y^{20}$
f. $\frac{625 x^{4}}{16 y^{4}}$
4. Mercury: $5.8 \times 10^{7}$

Earth: $1.5 \times 10^{8}$
Mars: $2.3 \times 10^{8}$
Saturn: $1.4 \times 10^{9}$
Pluto: $5.9 \times 10^{9}$
5. $1.4 \times 10^{12}$

## Transforming Equations Using <br> Multiple Operations

Student Logbook

1. $x^{2}$
2. $a x^{n}$; real number; variable; nonnegative
3. ploynomial
4. trinomial; 3 monomials or 3 terms
5. leff; right; descending order
6. leff; right; ascending order
7. nonzero value; simplify; identity
8. Their exponents are different.
9. Students' answers will vary.

## Your Turn

1. No. In the definition of a monomial, a term of the form $a x^{n}, \mathrm{n}$ is a nonnegative integer. Since $n$ in the expression $2 x^{-3}$ is negative, this expression is not a monomial.
2. a. $2 x^{2}+x$; binomial
b. $4 s^{23}-7 s^{17}-s$; trinomial
3. a. $2 x^{3}+10 x^{2}-2 x+12$
b. $-2 b^{4}-b+4$
c. $7 c^{3}+6 c^{2}-2$
4. a. $-3 a+11 a^{3}$
b. $3-x-6 x^{2}+8 x^{3}$
c. $b+3 b^{2}-b^{3}$
5. $\mathbf{a}$. $2 n^{2}+4 n$
b. $3 n^{2}+7 n+3$
c. $4 n^{2}+9 n+3$

## Multiplying Polynomials

Student Logbook

1. $n+1$
2. distributive; $(n+10) n+(n+10) \times 1$
3. sum; products
4. FOIL stands for First, Outer, Inner, Last: multiply the first terms of each binomial; multiply the outer terms; multiply the inner terms, multiply the last terms.
5. substitute; identity
6. $a^{2}+2 a b+b^{2}$
7. $a^{2}-2 a b+b^{2}$
8. $a^{2}-b^{2}$

## Your Turn

1. a. $(n+2)(n+8) ; n^{2}+10 n+16$
b. $n(n+8) ; n^{2}+8 n+0$
c. $(n-1)(n+8) ; n^{2}+7 n-8$
d. $(3 n+1)(n+8) ; 3 n^{2}+25 n+8$
2. $(n+3)(4 n-2) ; 4 n^{2}-2 n+12 n-6$; $4 n^{2}+10 n-6$
3. $a \cdot 9 b^{2}+12 b+4 ;(3(-2)+2)^{2}=9(-2)^{2}+$ $12(-2)+4$; $169=169$
b. $25 y^{2}-30 y+9 ;(5(-2)-3)^{2}=25(-2)^{2}$ $-30(-2)+9$; $169=169$
4. $x^{2}-16$
5. $2 n^{2}+16 n$; Sample answer. The area of one panel is $n(n+8)$ or $n^{2}+8 n$. This multiplied by 2 equals $2 n^{2}+16 n$.

## Unit Assessment

1. $b$ and $c$
2. a. $-32 a^{3}$
b. $5 \mathrm{r}^{6}$
c. $4^{-4}$, or $\frac{1}{4^{4}}$ or $\frac{1}{256}$
d. $16 y^{12}$
e. $8 s^{3 n+6}$
f. $\frac{64 r^{3}}{343 s^{3}}$
3. $3.8 \times 10^{-2}$
4. a. $7 n^{2}+10 n-5$
b. $-2 n^{2}+3 n+15$
c. $n^{2}-15 n-14$
d. $9 n^{2}+24 n+7$
5. $(2 n+3)(3 n-4)$
$=6 n^{2}-8 n+9 n-12$
$=6 n^{2}+n^{2}+2$
6. $d$
7. $(3-5)(3+5)=3-25$
$(-2)(8)=9-25$
$-16=-16$
8. a. $n^{2}-n$
b. $3 n^{2}+13 n+4$
c. $2 n^{2}+14 n+4$

## Unit Investigation*

1.Students' diagrams will vary. Dimensions should be reasonable.
2. Students' diagrams will vary.

Dimensions should be reasonable.
3. Students' diagrams will vary. Area and dimensions should be reasonable
4. Students' answers will vary.

### 2.2 Factoring Polynomials

## Finding Common Factors

## Student Logbook

1. positive integers; 1; itself
2. It has only one factor (itself).
3. composite number
4. prime factors; product
5. Factor the monomials. The greatest common factor of the variable terms is equal to the variable term with the lower exponent.
6. degree
7. $12 n^{2}$
8. the product of two or more polynomials
9. highest; monomial
[^0]
## Your Turn

1.a. $2 \cdot 2 \cdot 3 \cdot 5$ or $2^{3} \cdot 3 \cdot 5$
b. $5 \cdot 31 \cdot x \cdot y$
c. $2 \cdot 2 \cdot 2 \quad 2 \cdot 3 \cdot 3 \cdot n \cdot n$ or $2^{4} \cdot 3^{2} n^{2}$
2. a. $8 y^{3}$
b. $4 a^{3}$
3. $a$

b. $3(2 x+1)$
c. $3(4)(2(4)+1)=6(4)^{2}+3(4)$

$$
\begin{aligned}
& 12(8+1)=6 \cdot 16+12 \\
& 12 \cdot 9=96+12 \\
& 108=108
\end{aligned}
$$

4. a. $4 n\left(3 n^{2}+5\right)$
b. $8 y^{3}(9 y-5)$
c. $(x+2)(x+5)$
d. $3(m+7)(m+2)$

## Factoring Quadratic Trinomials

## Student Logbook

1. 24; 10
2. $(x+4)(x+6)$
3. quadratic term
4. linear term
5. constant term
6. Yes. It is a form $a x^{2}+b x+c ; a, b, c$ are 1, 10,24 which are real numbers and $a \neq 0$.
7. opposites
8. $(y-3)(y-4)$
9. $(2 r+3)(r+2)$
10. $(2 n+5)(3 n-2)$

## Your Turn

1.a.

b. $(g+2)(2 g+5)$
c. $2(2)^{2}+9(2)+10=(2 \cdot 2+5)(2+2)$
$8+18+10=9 \cdot 4$
$36=36$
2. a. $s^{2}+5 s-1$
b.s ${ }^{2}$
c. 5 s
d. -1
3. a. $(x+2)(x+3)$
b. $(d-8)(d+4)$
c. $(2 p+1)(p+3)$
d. $(3 y-4)(y-1)$
e. $3(f+2)(f-3)$

## Special Cases

Student Logbook

1. $(a+b)^{2}$
2. difference; two squares
3. $(2 x+3)(2 x-3)$
4. $(a+b)(a-b)$
5. $(5 k+12)(5 k-12)$
6. yes; $\left(x^{2}\right)^{2}=x^{4}$ and $8^{2}=64$
7. $\left(x^{2}-8\right)\left(x^{2}+8\right)$
8. prime
9. a. common factors; distributive
b. perfect square trinomial; difference of two squares
10. prime

## Your Turn

1. 

| Factored form | Trinomial expression | Special case |
| :--- | :--- | :--- |
| $(x+9)^{2}$ | $x^{2}+18 x+81$ | perefect square trinomial |
| $(2 x+10)^{2}$ | $4 x^{2}+40 x+100$ | perefect square trinomial |
| $(x-3)^{2}$ | $x^{2}-6 x+9$ | perefect square trinomial |
| $(x+5)(x-5)$ | $x^{2}-25$ | difference of two squares |
| $(x+7)(x-7)$ | $x^{2}-49$ | difference of two squares |
| prime, not factorable | $x^{2}+81$ | sum of two squares |
| $(2 x+20)^{2}$ or $4(x+10)^{2}$ | $4 x^{2}-80 x+400$ | perefect square trinomial |

2. a. $(a-b)^{2}$ is the square of a difference and is equal to $a^{2}-2 a b+b^{2} .\left(a^{2}-b^{2}\right)$ is the difference of squares and factors as $(a+b)(a-b)$.
b. Solutions will vary. Example: If $a=1$ and $b=2 ;(a-b)^{2}=(1-2)^{2}=1$ and $\left(a^{2}-b^{2}\right)=1^{2}-2^{2}=1^{2}=-3$

## Unit Assessment

1.The next step would be to eliminate all multiples of 3 from the table greater than 3 , then multiples of 5 greater than 5 , etc. The numbers remaining would not be multiples of any numbers preceding them, other than themselves and 1 , and are therefore prime.
2. $24=2 \cdot 2 \cdot 2 \cdot 3$ or $2^{3} \cdot 3$
3. a. Solutions will vary. Example: 8 and 12, GCF is 4
b. Solutions will vary. Example: 15 and 18, GCF is 4
4. $b^{4}$
5.

| Terms | Greatest common factor |
| :--- | :--- |
| 16,24 | 8 |
| $64 m, 32 m, 96 m$ | $32 m$ |
| $42 x^{2}, 18 x^{3}$ | $6 x^{2}$ |

6. a. $(x+3)(x+1)=x^{2}+4 x+3$
b. Solutions will vary. Example: Let $x=5$, $(5+3)(5+1)=5^{2}+4 \cdot 5+3$ $8 \cdot 6=25+20+3$ $48=48$
7. a. $(g-10)(g+2)$
b. $(k+6)^{2}$
c. $(p-8)(p+2)$
d. $(2 x+3)(2 x+5)$
e. prime
f. $(4 a-5)(4 a+5)$
8. a. $8(p+2)$
b. $4(p+4)$
c. $4(3 d+4)$
d. $8(x+2)(x-2)$

## Unit Investigation

1. 


2.

3. $c^{2}+4 c+4 ;(c+2)^{2}$
4. a.

b. $c^{2}+2 n c+n^{2} ;(c+n)^{2}$
5. a perfect square trinomial
6. Floor plans will vary.
7. Answers will vary. Example: If there is 6 feet of tile on one wall and 1 foot of tile on the other, then $x^{2}+7 x+6=(x+6)(x+1)$
8. The factored form of the area gives the dimensions of the room, and shows the length of the carpet region and the length of the tiled region.

### 3.1 Graphing Quadratic Functions \& Equations

## Graphing Parabolas

Student Logbook

1. a second degree polynomial function
2. $x$-value; $y$-value
3. up
4. down
5. the least value of $y$ on the graph of the parable
6. the greatest value of $y$ on the graph of the parable
7. axis of symmetry
8. $x=0$
9. vertex

## Your Turn

1. a. $a$; $c$
2. a. concave down
b. concave up
c. concave up
3. a. $b$ and $c$
b. $a$
4. $c$
5. Students' graphs should depict a parabola in quadrants I and II whose vertex is the origin.

The curve is concave up and should pass through points $(1,2),(-1,2),(2,8),(-2,8)$, $(3,18),(-3,18)$, and so on.
a. domain: $x=$ all real numbers; range: $y \geq 0$
b. $x=0$
c. the origin, $(0,0)$
d. concave up
e. $y=0$ (minimum)

## Analyzing Properties of Parabolas Student Logbook

1. the $y$-intercept
2. down; vertex; 1,000
3. 0 ; real numbers
4. incomplete; 0
5. midpoint; $h$
6. 240.1
7.7
7. parabola; $y$-axis; vertex
8. $x$-intercepts; 0

## Your Turn

1. a. -5
b. 0
c. 67
2. $b, c$
3. Students' parabolas should open upward, the least value of $y$ (the minimum) should be -3 , and the parabola should be centered around the axis of symmetry, $x=-2$. The parabolas may have any width, provided these conditions are met.
4. $(8,15)$
5. a. 400
b. the height of the cliff, or the distance between the cliff and the ground below it.

## Solving Quadratic Equations by Graphing

Student Logbook

1. trajectory
2. quadratic; linear; 0; -6
3. vertex
4. root; solution
5. horizontal intercepts
6. real roots
7.5.6
7. real root
8. $x$-intercept

## Your Turn

1.a. 5
b. 12 feet (at $d=10$ )
c.


Since $d$ cannot be negative in this problem, only the curve in Quadrant 1 and the prints $(0,5)$ and (25.0) correspond to the ball's trajectory.
2.

| Equation | Roots | Concavity | Max/min |
| :--- | :---: | :--- | :--- |
| $h=0,5 d^{2}+1$ | 0 | Concave up | 1 |
| $y=3 x^{2}+6 x+1$ | 2 | Concave down | 4 |
| $d=-1.9 t^{2}$ | 1 | Concave down | 0 |
| $y=4 x^{2}+4 x-35$ | 2 | Concave up | -36 |

## Unit Assessment

1. a. $3 x^{2}$
b. $5 x$
c. -7
d. The equation is of the form $y=a x^{2}+b x+c$.
e. the coefficient of the quadratic term, 3
f. The sign of the quadratic term. If it is positive, the parabola is concave up; if it is negative, the parabola is concave down.
g. the constant term, -7
2. a. maximum
b. minimum c. minimum
3. a. $y=x^{2}-1$
b. $y=-x^{2}+1$
4.a. 2 real roots
b. 2 real roots
c. 0 real roots
d. 1 real roots
4. 


6. $t=0$ to 5 inclusive
7. 0 to 70 inclusive

## Unit Investigation

l.a. and b.

c. $b_{2}$; The ball that was dropped hit the ground after 3.2 seconds, and the ball that was thrown hit the ground after 4.4 seconds.
d. About 56 meters
e. Between 1 and 1.5 seconds
f. 50 meters
2. a. The initial velocity (speed) of the ball when it was thrown.
b. The height of the cliff above the ground ( 50 ff .).
c. $0 \mathrm{~m} / \mathrm{s}$
3. $b_{1}=0$ and $b_{2}=44$
4.


| $b_{2}$ |  |
| :--- | :--- |
| $t$ | $d_{2}$ |
| 0 | 0 |
| 1 | 10 |
| 1.5 | 15 |
| 2 | 20 |
| 2.5 | 25 |
| 3 | 30 |
| 4.4 | 44 |

5. The ball fell at a constant speed: $10 \mathrm{~m} / \mathrm{s}$.

### 3.2 Solving Quadratic Equations Using Algebra

Factoring \& The Zero Product
Student Logbook

1. Zero product theorem
2. two
3. horizontal intercepts or $x$-intercepts
4. axis of symmetry, vertex
5. 0,0
6.1
6. 22.2
7. double root
8. $x$-intercepts; parabolic function

Your Turn
1.a. two
b. $0=0.25 x^{2}-4$

$$
\begin{aligned}
& 0=(0.5 x+2)(0.5 x-2) \\
& 0=0.5 x+2 \text { or } 0=0.5 x-2 \\
& -2=0.5 x \text { or } 2=0.5 x \\
& -4=x \text { or } 4=x
\end{aligned}
$$

2. $x(x+4)$
3. a. 0,2
b. $(1,3)$
C.

4. a. $(6 x+2)(6 x+2)$, or $4(3 x+1)^{2}$
b. $x=-\frac{1}{3}$
c. 1

## The Square Root Method \& Completing the Square <br> Student Logbook

1. square root property
2. $a+(-a)=0,-(a)+a=0$
3. sum
4. perfect square trinomial
5. $3 ;-13$
6. 4
7. $x-2=\sqrt{3}$ or $x-2=-\sqrt{3}$
(or $(x-2)^{2}=3$ )
8. square root property
9. completing; square
10. irrational

## Your Turn

1. two
2. a. $x= \pm 3$ b. $x= \pm 1 \quad$ c. $x= \pm 2$
3. $b^{2}$
4. a. 36
b. 100
c. $\frac{9}{4}$
5. $x^{2}+4 x-5=0$
$x^{2}+4 x=5$
$x^{2}+4 x+4=5+4$
$(x+2)^{2}=9$
$x+2= \pm 3$
$x+2=3$ or $x+2=-3$
$x=1$ or $x=-5$
6. $x^{2}-10 x+18=0$
$x^{2}-10 x+25=18+25$
$(x-5)^{2}=7$
$x-5= \pm \sqrt{7}$
$x=5 \pm \sqrt{7}$

## The Quadratic Formula

Student Logbook

1. $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
2. $2 ; 8 ;-13$
3. $2 \sqrt{42}$
4. real
5. $b 2-4 a c$
6. radicand
7. no real
8. one real
9. two real

## Your Turn

1.a. square root property
b. completing the square
c. zero product theorem
2. $g=\frac{-h \pm \sqrt{h^{2}}-4 f}{2 f}$
3. a. $x^{2}+12 x-18=0 ; 1 ; 12 ;-18$
b. $3 y^{2}-2 y+51=0 ; 3 ;-2 ; 51$
c. $8 y^{2}-2 y+27=0 ; 8 ;-2 ; 27$
d. $x^{2}+5 x-2=0 ; 1 ; 5 ;-2$
4. $x=-b \pm \sqrt{b^{2}-4 a c}$

$$
\begin{aligned}
& \frac{-(-5) \pm \sqrt{(-5)^{2}-4(5)(1)}}{2(5)} \\
& x=\frac{5 \pm \sqrt{25-20}}{10}
\end{aligned}
$$

$$
x=\frac{0.5+\sqrt{5}}{10} \quad \text { or } \quad x=\frac{0.5-\sqrt{5}}{10}
$$

## Unit Assessment

1. a. The equation has two solutions. This makes sense because he rabbit touches the ground $(x=0)$ at two points, the point that it jumps from and the point where it lands.
b. $x=0$ or $x=2$
c. distance $=2 \mathrm{ft}$.
d. $\frac{1}{4}$ of a foot or 3 inches
2. $m=9$ of $m=-9$
3. $s=0$ or $s=99$
4. $\left(\frac{b}{2}\right)^{2}$ or $\frac{b^{2}}{4}$
5. $x^{2}+18 x-19=0$

$$
x^{2}+18 x=19
$$

$$
\begin{aligned}
& x^{2}+18 x+81=19+81 \\
& (x+9)^{2}=100 \\
& x+9= \pm \sqrt{100}= \pm 10 \\
& x+9=10 \text { or } x+9=-10 \\
& x=1 \text { or } x=-19
\end{aligned}
$$

6. 


$x=\frac{-7 \pm \sqrt{49-20}}{2}=\begin{gathered}-7 \pm \sqrt{29} \\ 2\end{gathered}$
$x=-3.5+\frac{\sqrt{29}}{2} \quad x=-3.5-\frac{\sqrt{29}}{2}$
7. The discriminant tells you how many solutions here are; if they are real or not; and if they are real, if the are rational or irrational.
8.

| Quadratic equation | Discriminant | Nature of roots |
| :--- | :--- | :--- |
| a. $5 x^{2}+6 x+5=0$ | -64 | not real |
| b. $6 x^{2}+6 x+7=0$ | -132 | not real |
| c. $2 x^{2}+8 x+2=0$ | 48 | two irrational |
| d. $8 x^{2}+3 x-4=0$ | 137 | two irrational |

9. $a$ and $b$

## Unit Investigation

1. a. $h=-4.9 t^{2}+60$
b.

| $t$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | 60 | 55 | 29 | 16 | -18 |

c. 3 and 4 . The value of $h$, when $t$ is 3 , is positive. One second later, the value of $h$ is negative. So, somewhere between 3 and 4 , the value of $h$ must have been 0 , the value when the boulder strikes the ground.
d.

e. 60
f. Solve the equation for $t$ when $h$ equals 0 .
$-4.9 t^{2}+60=0$
$60=4.9 t^{2}$
$600=49 t^{2}$
$600=t^{2}$
$\frac{49}{10 \sqrt{6}} \frac{7}{7}=t$
$3.5 \stackrel{7}{\approx}$
2. a. 2 and 2 . the initial position of the ball occurs when $x=0$. therefore, since $y$ is the height in feet above each player's shoulder, when $x$ equals $0, y$ equals 2 in each equation.
b.

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y A$ | 2 | 6 | 8 | 8 | 6 | 2 | -4 |
| $y B$ | 2 | 6.7 | 8.8 | 8.5 | 5.6 | 0.3 | -7.6 |

c.

d. Player B
e. Player A: 26.9 and Player B: 25.2
f. Solve he equation when $y=0$.
$-0.04 x^{2}+x+2=0$
$x=\frac{-1 \pm \sqrt{1^{2}}-4(-0.04)(2)}{2(-0.04)}$
$x=\frac{-1 \pm \sqrt{1.32}}{-0.08}$
$x \approx \frac{-1 \pm 1.15}{-0.08}$
$x \approx \frac{2.15}{0.08} \approx \underline{26.9}$
g. Player A

### 4.1 Radical Equations \& Functions

## Solving Radical Equations

Student Logbook

1. radical equation
2. $a^{2}=b^{2}$
3. square root; $\frac{1}{2}$
4. radical;square
5. If the graphs of the functions intersect, then the $x$-coordinate of the point of intersection is the solution of the radical equation.
6. extraneous root
7. $x^{0.5}$
8. true
9. extraneous root(s)

## Your Turn

1. $x=25$
2. No. The symbol $\sqrt{m}$ indicates a principal, or non-negative root. Therefore $\sqrt{m}$ cannot be a negative.
3. $\sqrt{\bar{r}-5}-8=0$
$\sqrt{\bar{r}-5}-8+8=0+8$
$\sqrt{\bar{r}-5}=8$
$\left[(r-5)^{\frac{1}{2}}\right]^{2}=8^{2}$
$r-5=64$
$r=69$
Check: $\sqrt{69-5-8}=0$
$\sqrt{64}-8=0$
$8-8=0$
4. a. one solution:

b. $\sqrt{x+2}=x$
$x+2=x^{2}$ $0=+x^{2}-x-2=(x+1)(x-2)$ $0=x+1$ or $0=x-2$ $-1=x$ or $2=x$
c. $x=2$
d. $x=-1$

## The Inverse of the Square Root Function

## Student Logbook

1. one-to-one
2. inverse
3. interchange; function; inverse
4. $f^{-1}(x)$
5. range
6. domain
7. $y=x$
8. function; 2
9. $y= \pm \sqrt{x}$
10. domain

## Your Turn

1. Yes. For every point on the graph, each value of $y$. is paired with one and only one value of $x$, and each value of $x$ is paired with one and only one value of $y$.

## 2. a.

$f^{-1}(\mathrm{x})$

b. $x \geq 1$.
c. $f(x) \geq 0$.
d. $f^{-1}(x)=x^{2}+1$
e. $x \geq 0$.
f. See graph above
g. See graph above; $y=x$

## Unit Assessment

1. $\sqrt{a}=12 ;(\sqrt{a})^{2}=12^{2} . a=144$
$\sqrt{144}=12$
$12=12^{2}$
2. $\sqrt{k}+2=0 ; \quad \sqrt{k}=-2$. no solution
$\sqrt{(-2)}=2$
$\sqrt{(-2)^{2}}=2$
$-2 \neq 4$
3. $\sqrt{d-15}-5=0 ; \sqrt{d-15}=5$
$(\sqrt{d-15})^{2}=5^{2} \quad d-15=25 \quad d=40$
$\sqrt{4 \overline{0}-15}-5=0$
$\sqrt{25-5}=0$
$5-5=0$
4. $z=\sqrt{z+7}+5$
$z-5=\sqrt{z+7}$
$(z-5)^{2}=(\sqrt{z+7})^{2}$
$z^{2}-10 z+25=z+7$
$z^{2}-11 z+18=0$
$(z-2)(z-9)=0$
$z-2=0$ or $z-9=0$
$3=2$ or $3=9$
9 is a solution, and 2 is an extraneous root.
$2=1 \sqrt{2+7}+5$
$2=\sqrt{9+5}$
$2=3+5$
$9=\sqrt{9+7}+5$
$9=\sqrt{1 \overline{6}+5}$
$9=4+5$
5. one solution; $x=1$

6. $\sqrt{x}-1-1=\frac{x}{5}$
$\sqrt{x}-1=\frac{x}{5}+1$
$(\sqrt{x}-1)^{2}=\left(\frac{x}{5}+1\right)^{2}$
$(x-1)=\frac{x^{2}}{25}+\frac{24}{5}+1$
$25-25=x+10 x+25$
$0=x^{2}-15 x+50$
$0+(x-5)(x-10)$
$0=x-5$ or $0=x-10$
$5=x$ or $10=y$
7. No; It is not a one-to-one function because each non-zero value of $y$ is paired with one value of $x$.
8. a. The domain is $x \geq-2$.
b. The range is $y \geq 0$.
c. $f^{-1}(x)=x^{2}-2$
d. $x \geq 0$
e. $f^{-1}(x) \geq-2$
f.


## Investigating Gravity

## Student Investigation

1. The rock is 192 feet above the ground.

$$
t=\sqrt{16-\frac{1}{16} d} \quad 2=\sqrt{16-\frac{1}{16} d}
$$

$(2)^{2}={\sqrt{16-\frac{1}{16}}{ }^{2}}^{2}$
$4=16-\frac{1}{16} d$
$-12=-\frac{1}{16} d$
$192=d$
2. No. After 3 seconds the rock is 112 feet above the ground. Since the bird is flying at a height of 120 feet, it will be above the rock.
3.

4. $0 \geq d \geq 256$
5. 2.8 seconds

$$
d=\frac{256}{2}=128
$$

$t=\sqrt{16-\frac{1}{16}(128)}$
$=\sqrt{16}-\frac{128}{16}$
$=\sqrt{16-8}$
$=\sqrt{8}$
= $2.82 . . . \approx 2.8$
6. The rock hits the ground when $d=0$ :
$t=\sqrt{16-\frac{1}{16} d}$
$=\sqrt{16}-\frac{1}{16}(0)$
$=\sqrt{16}=4$
So, the rock hits the ground after 4 seconds.
7. Since the domain of the function is $0 \leq \mathrm{s} \leq$ 256 , the range of the function is $0 \leq t \leq 4$.
8. Yes. Answers may vary, but students should indicate that the function is a one-to-one function because in this domain, for every value of $d$ there is one and only one value for $t$.

### 4.2 Rational Expressions, Equations \& Functions

## Rational Operations

## Student Logbook

1. rational expression; one
2. $\frac{a}{c} \times \frac{b}{d}$
3. $\frac{a}{0}$
4. excluded value
5. Yes; the quotient of two rational expressions can be expressed as the product of two rational expressions.
6. $\frac{a+c}{b}$
7. $\frac{a d \pm c b}{b d}$
8. denominator; $x$
9. $\frac{d}{b}$

Your Turn

1. a. -2
b. 0
c. -1
d. $\pm 2$
2. a. $b \neq-2,0,2 ; \frac{5}{b(b-2)}$
b. $c \neq 0 ; \frac{4 c+12}{c}$
c. $d \neq 0 ; \frac{2-5 d}{4 d^{2}}$
d. $h \neq 0,6 ; \frac{h+6}{h(h-6)}$
e. $k \neq 0,2 ; \frac{5 k-14 k+2}{k(k-2)}$

## Rational Functions

## Student Logbook

1. rational equation
2. hyperbola
3. vertical axis, horizontal axis
4. $\rightarrow+\infty \quad \rightarrow-\infty$
$\rightarrow 0^{+} \quad \rightarrow 0^{-}$
5. asymptote
6. discontinuous
7. excluded value, vertical asymptote
8. horizontally
9. asymptotes

## Your Turn

1.a. $>$
b. $<$
c. $>$
d. $<$
2. $x=1$
3. $a$.
b.


## Rational Equations

## Student Logbook

1. common denominator
2. $x=3$
3.3
3. rate, time, work
4. $7 x$
5. $t=\frac{d}{r}$
6. extraneous root
7. excluded values
8. rational equation

## Your Turn

1. a. $2 k^{2}+8 k$ or $2 k(k+4)$
b. $12 w^{2}-36 w$ or $12 w(w-3)$
c. $p^{2}-5 p+6$ or $(p-3)(p-2)$
2. $p=2.8$
3. $b=3$
4. $3 \frac{1}{3}$ hours
5. a. $\frac{r t}{t-30}$
b. $8: 10 \mathrm{am}$
$6.3 \frac{1}{3}$ minutes

## Unit Assessment

1. a. $h \neq 0$
b. $g \neq-3$ c. $d \neq 2$
2. a. $\frac{2(w+3)}{w-2}$
b. $\frac{2 z^{2}+5 z-3}{6 y z^{2}-12}$
c. $\frac{4-\mathrm{u}}{36}$
d. $-y^{2}=5 y+42$
3. 

| $\mathbf{x}$ | $\boldsymbol{f}(\mathbf{x})$ |
| :---: | :---: |
| -4 | -2 |
| -3 | undefined |
| -2 | 2 |
| -1 | 1 |
| 0 | 2 |
| 1 | $\frac{2}{2}$ or ${ }_{2}$ |
| 2 | 2 |
| 3 | $2_{5}^{2}$ or ${ }_{3}^{1}$ |
| 4 | $\frac{2}{7}$ |

4. a.

b.

5. $\frac{t}{6}+\frac{t}{h}=1$, so if $t=4, h=12$ hours
6. a. 5 miles
b. 18 mph
a. 1.5 hours

## Unit Investigation

1. 



2. It decreases.
3. It decreases.
4. $\frac{1}{2}$ swing per second
5. $\frac{1}{4}$ seconds
6. The graph is the part of the hyperbola in quadrant 1, and translated farther from the $y$-axis than the other 2 graphs
7. Increasing the wavelength of a wave causes the period of a wave to increase for a given speed.

### 5.1 Graphical Displays

## Stem \& Leaf Plots \& Box Plots <br> Student Logbook

1. collection; display; analysis
2. histogram; intervals
3. stem; leaf
4. difference
5. skewed
6. median
7. four
8. second quartile
9. box plot or box-and-whisker plot
10. third; first
11. outlier

## Your Turn

1. 

| stem | leaf |
| :--- | :--- |
| 3 | 3 |
| 4 | 29 |
| 6 | 57 |
| 10 | 57 |
| 28 | 7 |
| 29 | 7 |
| 53 | 4 |

Key: $3 / 3=3.3 \%$
2. a. 3.3
b. 53.4
c. 15.86
d. 8.6
3. $4.9 ; 8.6 ; 28.7$
4. 23.8
5.

6. 53.4

## Scatter Plots \& Linear Best-Fit Graphs

Student Logbook

1. two
2. best-fit
3. median-median
4. No; the line $M_{1} M_{3}$ does not represent the data in the center group because most of the data points in the center group lie below the line $M_{1} M_{3}$
5. parallel $; \frac{1}{3} ; \frac{2}{3}$
6. trend
7. predictions

## Your Turn

1. 


2. $\mathrm{M}_{1}=(1932,58.2)$
$M_{2}=(1960,55.2)$
$M_{3}=(1980,49.99)$
3. a. $y=-0.171 t+388.57$
b. $y=-0.171 t+390.36$
c. $y=-0.171 t+389.167$
4. $s=48.535$

## Unit Assessment

1. 

| stem | leaf | $2 \mid 5=25 \mathrm{mph}$ |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 5 |  |  |  |
| 3 | 0 | 2 | 2 |  |
| 4 | 0 | 2 | 3 | 5 |
| 5 | 0 |  |  |  |
| 7 | 0 |  |  |  |

a. 40.9 mph
b. 70 mph
c. 41
d. Yes; because the mean is nearly equal to the median.
2. a. $32,41,45$

b. 13
c. The interquartile range shows that approximately $50 \%$ of the animals listed in the table have speeds between 32 mph and 45 mph .
3. a. one-dimensional
b. one-dimensional
c. two-dimensional
4. a. $M_{1}=(1937.5,64.55)$ $M_{2}=(1960,73.1), M_{3}=(1982.5,77.8)$
b. $m=0.294$
c. $y=0.294 x-504.43$
d. 82.1 years

## Unit Investigation

1. A box plot is te best way to display the desired information. The range of years that
the middle $50 \%$ of the parks have been established is the interquartile range. To find this range, each quartile must be found.
$Q_{1}=\frac{28+35}{2}=31.5$
$Q_{2}=\frac{70+75}{2}=72.5$
$Q_{3}=\frac{80+84}{2}=82.0$
The interquartile range is $Q_{3}-Q_{1}$ or $82.0-31.5=50.5$.

Outliers include Saguaro (5 years) and Yellowstone ( 127 years).

2. A stem-and-leaf plot is the best way to display the desired information. The maximum is 127 years, and the minimum is 5 years. The mean of the data is 61.8 . There are 5 values below the mean and 7 values above the mean. Thus, the data are only slightly skewed, and the mean does represent a good measure of central tendency.

| stem | leaf | $2 \mid 8=28$ years |
| :--- | :--- | :--- |
| 0 | 5 |  |
| 2 | 8 | 8 |
| 3 | 5 |  |
| 7 | 05 |  |
| 8 | 0 | 0 |
| 9 | 3 |  |
| 12 | 7 |  |

3. A scatter plot is the best way to see if there is a general trend in the data. Accept all reasonable predictions for 2000. Sample: The prediction may not be reasonable because during the last ten years, the number of visitors has fluctuated.


Equation of $M M$ line is $y=1,030.9 x-1,993,229.8$


[^0]:    *This problem is appropriate for group work, as well as individual assignments.

